The Open University

S102PRP/S510 WORKBOOK INTO SCIENCE

WORKBOOK

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2.1

STUDY GUIDE

The purpose of the Workbook is two-fold: to provide additional practice in mathematical skills and to improve your understanding of some basic science. Thus it is full of questions and problems for you to answer and solve. By becoming *fluent* in the skills practised in the Workbook, your study of further science courses will be much easier.

The wide range of questions on each topic allows you to explore ideas more thoroughly than is possible in the SAQs in the Modules alone. The comments in the ANSWERS part of the Workbook provide some useful hints as well as extra teaching points. Occasionally, questions and answers go a step further than the examples given in the Modules. In addition, there are further hints on how to use your calculator. Also there are many problems couched in the *words* and in the language of science. Being able to tackle these types of problem is important in science: mathematics is required in real situations, and not just 'bare' as in maths textbooks. Coping with the words, that is understanding what is required, is usually more than half the battle!

However, a word of warning may be helpful! It is possible to misuse the Workbook—there is just *too much* in the nine Sections that follow for you to do *all* the exercises. And, anyway, once you know that you can do something accurately and fluently, there is not a lot to be gained in practising it needlessly. The questions range in level of difficulty. This is because different readers have different strengths and weaknesses—and different likes and dislikes. Thus, some of the obviously difficult questions might be interesting and challenging to you whereas other questions might be just too easy or too difficult! Be realistic and tackle the right *amount* of the right *kind* of material to suit YOU. We do not expect you to become expert in all the skills introduced in *Into science*!

As you can see from the 'Contents' list, the nine sets of QUESTIONS come before the corresponding nine sets of ANSWERS AND COMMENTS. Like the Modules themselves, the sets of questions get progressively more difficult. If necessary, use the ANSWERS AND COMMENTS to *teach yourself* how to do a question. Remember, too, that there is often more than one way to solve a problem; as long as you arrive at the correct answer, your method is fine. Where relevant, your answer must be in the right units, for example, kilograms or milligrams. Once you have learnt about significant figures in Module 2, you need to take care to give your answers to the correct number of significant figures.

You need to find which way of using the Workbook suits you best. We suggest one possible strategy: each week, glance through the QUESTIONS relevant to the Module that you are currently studying, just to see what is covered. Then, when you have finished the Module—or during your study of it—use the Workbook for whatever practice you need and have time for. Remember, the suggested study times for each Module given in Table 1 of Module 1 *do not* include time spent on the Workbook!

Concepts later in the Course depend on skills built up in earlier Modules. In general, before you move on to the algebra, geometry and trigonometry in the end part of *Into Science* (Modules 8 to 10) you need to have got to grips with the key mathematical skills in Modules 1 to 4. So, you should aim to be comfortable with the basic functions on your calculator (Module 1), feel easy about decimals (Modules 1 and 3) and significant figures (Modules 2 and 3), and have a reasonable idea about the way of writing numbers in 'scientific notation' (Module 4). If you meet a stumbling block on any of these points, you need to conquer the difficulty before moving on. Use the Workbook to do that.

You will need paper, pencils and pens and your calculator to hand. Module 1 explains the calculator keys you need to use in *Into Science*. You should note that the names on these keys vary between makes. Note also that when working with large numbers, you might find that your last digit varies slightly from ours. Finally, a number of abbreviations are used throughout the Workbook. These are given in the key so that you can easily refer to them.

Key for abbreviations

Meaning	Abbreviation
significant figures	sf
therefore	$\therefore$
because	$\because$
approximately	$\approx$
not equal to	$\neq$
per cent	%
proportional to	$\propto$
angle	$\angle$
experiment	expt
number	No. or no.

MODULE 1 QUESTIONS

Q1 Use your calculator to work out (to evaluate) the following:

(a) 103×11 (b) $5 + 791 + 263$ (c) $6\,998 - 993$
(d) $40\,584 \div 89$ (e) $98 - 48 + 4 + 8 - 10 - 15$

Q2 Rewrite these numbers in the conventional 'group of three' way and then say them aloud.

(a) 11001 (b) 976543 (c) 1000000 (d) 100000120
(e) 2000000000

Q3 (a) A tray has spaces for 12 eggs. A girl puts one rice grain in the first space, two in the second, three in the third and so on until she has put 12 in the twelfth space. How many grains are in the tray altogether?

(b) If she started with 858 rice grains, how many has she left after she has filled the tray in this way?

(c) How many more empty trays could she fill in this way (from the rice she has left)?

Q4 Use your calculator to evaluate the following:

(a) $12 \times 6 + 8$ (b) $8 + 6 \times 12$ (c) $12 \times 2 \times (6 + 8)$
(d) $8 + 6 \times 5 - 2$ (e) $(8 + 6) \times (5 - 2)$ (f) $3(6 + 1)$

Q5 Use your calculator to evaluate the following:

(a) $14(5 - 3)$ (b) $(97 - 8) \times (16 + 4)$
(c) $(26 - 2) \div 4$ (d) $36 - 12 \div 4$
(e) $36 - (12 \div 4)$ (f) $(3 \times 24) \div (4 \times 2)$
(g) $3 \times (24 \div 4) \times 2$ (h) $3 \times 24 \div 4 \times 2$

Q6 Use your calculator to evaluate the following:

(a) $\dfrac{3 \times 24}{4 \times 2}$ (b) $3 \times \dfrac{24}{4} \times 2$

Q7 On average, a mouse eats 5 g protein, 15 g carbohydrate and 5 g fat per day and a rat eats 7 g protein, 20 g carbohydrate and 3 g fat. If a laboratory houses 12 mice and 20 rats, what total mass of food is eaten (a) per day, (b) per week? (c) What fraction of this total is consumed by the mice?

Q8 Evaluate to two decimal places:

(a) $3.13 + 6.91$ (b) 0.9×0.1 (c) 0.091×10
(d) 0.0912×100 (e) $4.71 \div 2.1$ (f) $10 \div 6$

Evaluate to three decimal places:

(g) 8.190×7.55 (h) $0.9 \div 0.7$
(i) $1.4 \div 1.576$ (j) $2.00 - 0.6666$

Q9 The apparatus in Figure 1 was used in an investigation into breathing.

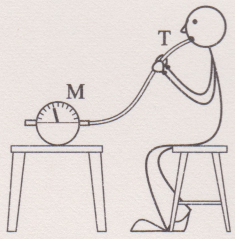


FIGURE 1 Apparatus used to investigate breathing.

John breathes through tube T and the volume breathed in or out is measured on meter M. When John is relaxed and breathing normally, he takes in 0.5 litres at each breath in and exhales 0.5 litres at each breath out. His breathing rate is 12 inhalations and exhalations per minute.

When instructed to breathe in *as deeply as he can*, he first inhales the normal 0.5 litres, and then in the same breath manages to breathe in a *further* 2.1 litres. Later, when he is once again breathing normally, he is instructed to breathe out *as far as he can*. To do this he first exhales his normal 0.5 litres, and then manages to breathe out a further 1.3 litres of air.

From information in a textbook, Mary learns that all people have a 'residual lung volume' and this is the volume of air left in the lungs that remains no matter how hard a person breathes out. She estimates (from the information in the book) that John's residual volume is 1.5 litres.

(a) When breathing quietly, what volume of air does John breathe in over a one hour period?
(b) What is the *total* volume (to one decimal place) of air in John's lungs when he holds his breath after breathing in as far as possible? Note that 'total volume' includes 'residual lung volume'.
(c) How many times larger is this *total* volume than the normal amount he breathes in (in each breath)?

Q10 25.0 g of grass was dried until no moisture remained. Its dry mass was 4.1 g. Calculate the following to one decimal place.

(a) What mass of water was there in 25.0 g of grass?
(b) What mass of water would have been in 100 g grass?

Q11 Convert the following fractions to decimals (to 3 decimal places):

(a) $\frac{1}{12}$ (b) $\frac{2}{15}$ (c) $\frac{11}{1000}$

Q12 Evaluate the following. Remember brackets around individual numbers are included only for easy reading. They do not have to be entered into the calculator. Other brackets, if any, must be entered.

(a) $(-4) + (-6)$ (b) $(-4) + 24$ (c) $(-19) - (-53)$
(d) $(-44) + (-101)$ (e) $(-16) \times (-5)$ (f) $(-107) \times 12$
(g) $(-98) \div 7$ (h) $(-4\,970) \div (-35)$

Evaluate to 2 decimal places:

(i) $144.6 \times (-1.27)$ (j) $(-0.11) - (-4.98)$
(k) $(-4.55) - (-6.887)$

Q13 A scientist notes that her laboratory deep freeze is 45 °C cooler than the laboratory air temperature. The freezer thermometer registers − 31 °C. What temperature is her lab?

MODULE 2 QUESTIONS

Q1 Convert the following into metres:

(a) 42.6 km (b) 0.0013 km (c) 970 mm (d) 47.58 mm.

Q2 In every minute the tip of the minute hand of a church clock moves a distance of 120 mm. Calculate how far the tip travels in 365 days, expressing your answer in appropriate units of distance.

Q3 The stem of a plant grows in length by 0.816 m in exactly 40 days. Calculate (a) how many millimetres it grows in 40 days and (b) how many mm the stem grows in an hour.

Q4 A man on a strict diet notes his daily intake of food in kilocalories. Over a five day period he ate the following amounts each day:

975 kcal, 1040 kcal, 1500 kcal, 785 kcal, 750 kcal

Calculate his average daily intake.

Q5 A woman swam across a lake and back again 14 times in all. This took her exactly 70 minutes. The width is 40 metres. What was her average speed of swimming?

Q6 The masses of seven stick insects are given below.
1.60 g, 1.50 g, 1.70 g, 1.75 g, 1.71 g, 1.75 g, 1.69 g

What must be the mass of an eighth stick insect if the average (of all eight) is 1.75 g?

Can you say anything about the mass of the eighth insect even before you begin your calculation?

Before you begin the following, check your understanding from Module 2 of the phrase 'to two significant figures'. Recall from Module 1 the meaning of 'to two decimal places'.

Q7 To how many decimal places are the following expressed?
(a) 14.170 (b) 3 (c) 3.0 (d) 1.596 7 (e) 0.1

Q8 To how many significant figures (sf) are the numbers in (a) to (e) of Q7 expressed? (*Note: sf is used for significant figures from here onwards in this Workbook.*)

Q9 Express the following to 3 sf:
(a) 1.752 (b) 2 101.2 (c) 1.055
(d) 475.491 (e) 21.450

Q10 The time taken for a journey is the distance travelled divided by the average speed. If a car travels at an average speed of 86 km hr^{-1} for a journey of 248 km, how long does the journey take in hours and minutes?

Q11 Convert these fractions into their lowest terms:
(a) $\frac{9}{12}$ (b) $\frac{15}{20}$ (c) $\frac{7}{21}$ (d) $\frac{5}{13}$ (e) $\frac{16}{48}$

Q12 Two varieties of butterfly, brown and white, were collected from a wood. Numbers were:

Males		Females	
Brown	White	Brown	White
70	10	75	15

Calculate the following. Ratios should be expressed in whole numbers in their lowest terms.
(a) the ratio of males to females
(b) the ratio of females to males
(c) the ratio of brown to white butterflies
(d) the ratio of white to brown butterflies
(e) the ratio of brown males to brown females
(f) the ratio of white males to white females
(g) the ratio of brown males to all butterflies
(h) the percentage of females (to 2 sf)
(i) the percentage of males (to 2 sf).

Q13 Suppose the ratio of the number of petrol cars to the number of diesel cars is 7 : 1. To the nearest whole number, how many petrol cars would you expect to find in a car park containing exactly 500 cars?

Q14 The width of a rectangular piece of metal sheeting was 40% of its length. The width was 80 mm.
(a) What was its length?
(b) What was the ratio of length to width?

MODULE 3 QUESTIONS

Q1 Convert the following units of length as shown:
(a) 12.2 m into cm (b) 491 mm into cm
(c) 14 cm into m (d) 95 cm into mm
(e) 1.97 m into cm (f) 110 cm into m.

Q2 Calculate the areas of the following rectangles. Express each answer in units that correspond to the question e.g. if the sides are given in mm, give the area in square millimetres (mm^2).
(a) 6.0 mm long × 10 mm wide (b) 4.2 mm × 9.1 mm
(c) 2 m × 10 m.

Q3 Calculate the areas of the following rectangles. Express each answer in the units shown in brackets.
(a) 1 m × 10 cm (m^2) (b) 1 m × 10 cm (cm^2)
(c) 200 cm × 14 mm (cm^2) (d) 200 cm × 14 mm (mm^2).

Q4 A garden is 7.5 m wide and exactly 5 m long. (a) Calculate its area in m^2 to one decimal place. (b) Calculate its area in mm^2. (c) Which unit is it sensible to use and why?

Q5 Fill in the blanks in the table below, making use of your answers to Q3 and Q4. Check your answer to the 'cm^2 to mm^2 conversion' by using Figure 2.

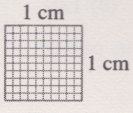


FIGURE 2 A square cm.

mm^2 = ...cm^2

1 cm^2 = ...mm^2

1 m^2 = ...mm^2

Q6 Convert the following units of area:
(a) 2 m^2 into cm^2 (b) 3 m^2 into mm^2
(c) 900 mm^2 into cm^2 (d) 16 000 000 mm^2 into m^2
(e) 250 000 cm^2 into m^2.

Q7 A biology teacher is trying to give her students a feeling for the very large internal surface area in human lungs (around 90 m^2 of wet surface across which oxygen and carbon dioxide diffuse). She decides to use floor area to illustrate her point. 'Your lungs have an internal area of about two and a half times the area of this classroom'. If the width of the room was exactly 5 metres, how long was it?

Q8 A rectangular floor in an excavated Roman villa consists of 719 400 square tiles of side 1.20 cm. The length of the floor is 13.08 metres. How wide is it?

Q9 Calculate the volumes of the following rectangular solids. Express each answer in units that correspond to the question, e.g. if the sides are given in mm, give the volume in cubic millimetres (mm^3). Ignore the issue of significant figures.
(a) 10 mm × 11 mm × 12 mm
(b) 1 cm × 1.1 cm × 1.2 cm
(c) 2 m × 5 m × 9 m
(d) 200 cm × 500 cm × 900 cm.

Q10 (a) Compare your answers to parts (a) and (b) of Q9. How many mm^3 = 1 cm^3?
(b) Compare your answers to parts (c) and (d) of Q9. How many cm^3 = 1 m^3?
(c) Using your answers to (a) and (b) of this question, how many mm^3 = 1 m^3?
(d) How many billion mm^3 equal 1 m^3?

Q11 Calculate to 2 sf the volumes of the following rectangular solids. Express each answer in the units shown in brackets.
(a) 2.0 m × 8.0 cm × 9.0 mm (cm^3)
(b) 87 cm × 9.0 cm × 8.0 m (m^3).

Q12 A and B are rectangular fish tanks. The base of A is 60 cm × 1.0 m; that of B is 1.2 m × 1.5 m. The two tanks are connected by a rubber tube and positioned as shown in Figure 3. Clip C is opened and tank A begins to drain into tank B. The level in A falls by 12 cm in the first minute. Calculate by how much (in cm) the level in B rises in the first minute. (Hint: first calculate what *volume* flows out of A into B in the first minute).

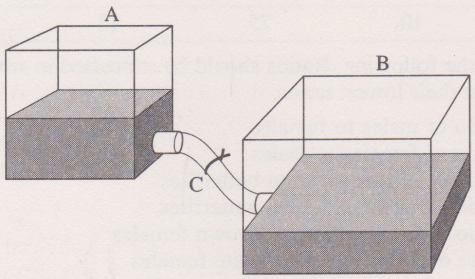


FIGURE 3 Water draining from tank A to tank B.

Q13 A balance was used to measure the masses of some cubes (all of side 2.000 cm) of different metals. The results are shown in the Table below. Calculate the densities of the metals (in g per cm^3) and enter them in the Table.

metal	mass/g	density/ g per cm^3	metal	mass/g	density/ g per cm^3
(a) aluminium	17.36	...	(b) gold	154.4	...
(c) copper	71.44	...	(d) lead	90.72	...

Can you tell which metal has the greatest density without any calculation? Explain your answer.

Q14 A solid will float on a liquid if its density is less than that of the liquid (Module 3, Section 4.4). The mass of 20.0 cm^3 of the metal mercury (a liquid at room temperature) was found using a balance and a glass beaker. Results were:

mass of beaker = 143.0 g

mass of beaker + mercury = 414.8 g

Subtraction of the first mass from the second gives the mass of mercury.

(a) Calculate the density of mercury in g per cm^3.
(b) Which of the metals in Q13 would not float on mercury? Explain your answer.

Q15 Convert these densities in g per cm^3 to kg per m^3:

substance	density/ g per cm^3	substance	density/ g per cm^3
(a) bone	1.9	(c) air	0.001 3
(b) ice	0.90	(d) helium	0.000 18

Using information in the Table, explain why helium is used to fill fair-ground balloons.

Q16 The lengths of the sides of six triangles are listed below. Pythagoras' theorem is obeyed only by right–angled triangles. Which two triangles are *not* right–angled?

(a) 1 cm, 2 cm, 3 cm (b) 3 cm, 4 cm, 5 cm
(c) 4 cm, 7.5 cm, 8.5 cm (d) 4.2 cm, 5.6 cm, 7 cm
(e) 5 cm, 12 cm, 13 cm (f) 6 cm, 7 cm, 9 cm.

Q17 Any rectangle can be divided by a diagonal line into two identical right–angled triangles (shown in Figure 4).

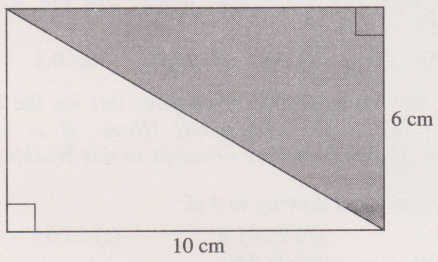


FIGURE 4 A rectangle is made up of two right-angled triangles.

Clearly, the *area* of each triangle is HALF that of the rectangle. So, in general terms we can say that the area of any right-angled triangle is:

$\frac{1}{2}$ × (length of its horizontal × its perpendicular height)

Given this information, work out the area of the shaded triangle in Figure 4.

Q18 A string of triangular flags is to be stretched from one side of a street to the other. The flags form right-angled triangles with two equal sides of 10 cm, as shown in Figure 5. The string is absolutely horizontal and taut and the walls are 20 m from each other.

Calculate how many flags can be fitted along the 20 m string without any overlap.

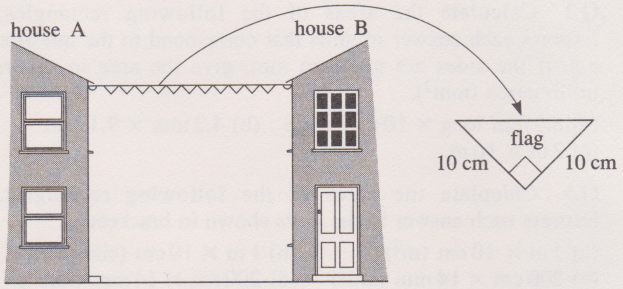


FIGURE 5 A string of flags and one flag enlarged.

MODULE 4 QUESTIONS

Q1 Express the following numbers in scientific notation:
(a) 800 (1 sf) (b) 800 (3 sf) (c) 1 239.0 (d) 1 230.0

Q2 The following numbers are in scientific notation. Write them in a manner which does *not* involve powers of ten, indicating the number of sf to which your answer is expressed: e.g. 1.00×10^2 is 100 to 3 sf
(a) 1.4×10^3 (b) 1.400×10^3 (c) $2.493\,1 \times 10^2$.

Q3 Express the *numbers* in the following statements in scientific notation (to 2 sf): (a) the total area of sea on the planet Earth is about 360 million km^2 (b) the distance from the Earth to the Sun is about 150 billion metres.

Q4 Express the following numbers in scientific notation:
(a) 0.06 (b) 0.080 (c) 0.000 024 0 (d) 10.0 (e) 1.01

Q5 Convert the following numbers, already in powers of ten, into scientific notation:
(a) 183.6×10^4 (b) 12.2×10^6
(c) 19.4×10^{-4} (d) 0.02×10^{-12}.

Q6 Without a calculator, and ignoring significant figures, do the following subtractions and additions and express the answers in scientific notation:
(a) $1.3 \times 10^6 - 1.4 \times 10^5$ (b) $6 \times 10^9 - 2 \times 10^7$
(c) $10^4 - 10^3$ (d) $2 \times 10^2 - 3 \times 10^{-1}$
(e) $2.2 \times 10^{-3} + 5 \times 10^{-2}$ (f) $2 \times 10^4 + 10^3$.

Q7 This time using your calculator, redo Q6 (a) to (f). Before you begin, satisfy yourself that you can enter powers of ten on your calculator by doing the following.

Enter 1×10^2 on your calculator. Recall that you do this by entering 1 then EXP (or EE) then 2. The display *looks* as if it is showing 1^2 or 1^{02} or 1. 02 but this is simply the way calculators display 1×10^2. Prove this to yourself by entering ÷, 2, and then the = button. The value of 50 that you get proves it was 100 (1×10^2) that you entered in the first place!

Q8 In the blood of an adult man there are about 4.5×10^{10} white cells and 3×10^{13} red cells. Use your calculator to find the total number of blood cells. (Ignore sf in answering this question.)

Q9 Without using a calculator, express the following using single powers of ten. (Ignore sf in answering this question.)

(a) $10^4 \times 10^{10} \times 10^{-2}$ (b) $10^{-2} \times 10^{-3}$ (c) $10^4 \times 10^{-4}$
(d) $10^3 \times 10^{-2}$ (e) $3 \times 10^{13} \times 5.5 \times 10^9$.

Q10 Without using a calculator, express the following using single powers of 10:

(a) $10^3 \div 10^3$ (b) $10^2 \div 10^{-3}$
(c) $(6 \times 10^4) \div (3 \times 10^2)$ (d) $10^9 \div 10^3$
(e) $10^9 \div 10^{-3}$ (f) $3 \times 10^3 \div 2 \times 10^2$
(g) $10^9 \times 10^2 \div 10^{-4}$ (h) $10^{-7} \times 10^6 \div 10^{-10}$.

Check that you can do these *with* a calculator, too.

Q11 In an adult man there are about 3×10^{13} red blood cells. His total blood volume is about 5 litres. Use your calculator to find the number of red blood cells per microlitre (µl) of blood. Note that 1 µl = 10^{-6} litres.

Q12 Use your calculator to do the following: 1 000 cm^3 of water (= 1 litre) contains about 10^{25} molecules of water.

(a) How many molecules of water are there in 1 m^3?
(b) How many molecules of water are there in 1 µl (1 µl = 10^{-6} l)?

Q13 The first cell from which all humans grow is called a zygote (pronounced 'z'—as in zoo—then 'eye' then 'goat'). Each person's zygote contains all the genetic information that is needed for him or her to become a unique adult. The substance that contains this genetic message is called DNA. Each human zygote contains just 6.80×10^{-12} grams of DNA—that is, the genetic message for an entire person is coded in seven million millionths of one gram of DNA!

Given that there are about 5.50×10^9 humans alive at present, what mass of DNA contained the code for all people currently alive on the planet?

Q14 The speed of sound in air at sea level and at a temperature of 15 °C is about 340 m s^{-1}. A man fires a pistol on the deck of a ship anchored 1.1 km from a cliff. After how many seconds will he hear the echo of the sound of the shot reflected from the cliff? (Hints: remember that the sound has to get to the cliff and back again, and remember to work in just *one* kind of unit of distance.)

Q15 Recall that the reciprocal of a number is 1 divided by that number, e.g. the reciprocal of 4 is 1/4 (a quarter) and the reciprocal of 1/4 is 4. On your calculator either divide 1 by the number or, much quicker, enter the number and press the 'reciprocal button' (usually shown as $1/x$). Reciprocals of ten raised to a power (e.g. 10^2) can be taken either by calculator or by just changing the sign of the power. To find the reciprocal of a number written in scientific notation, e.g. 4×10^3, you take the reciprocal of each part and then multiply them together, e.g.:

$1/4 \times 10^{-3} = 0.25 \times 10^{-3} = 2.5 \times 10^{-4}$. A calculator does it automatically: enter 4 EXP 3 and press $1/x$.

Write down (or use the calculator as necessary) the reciprocals of the following (ignore sf in answering this question):

(a) 0.25 (b) 3/8 (c) 1/1 000 (d) 10^{-3}
(e) 10^9 (f) 2×10^3 (g) 5×10^{-6}.

Q16 The star Sirius is 8.2×10^{17} m from Earth, whereas the Great Nebula in Andromeda is 2.1×10^{22} m from Earth.

(a) How many times further from Earth is the Andromeda galaxy than the star Sirius? (2 sf)
b) If a space ship travelled at the speed of light (3.0×10^8 m s^{-1}), how many years would it take to travel from Earth to Sirius? (2 sf)

MODULES 5/6 QUESTIONS

Q1 Among other substances, drinking vessels can be made of (a) glass (b) pewter (c) pure gold. Given that glass consists of silicon dioxide and pewter contains atoms of tin and lead which are not chemically combined together, give the following for each of (a), (b) and (c):

(i) the atoms present
(ii) the uncombined elements
(iii) elements present in compounds.

Q2 Which of the following statements when applied to one atom of nitrogen are correct? (Note: more than one statement is correct.)

(a) Its diameter is around 10^{-10} m.
(b) It contains the same number of protons as electrons.
(c) Its mass is 14 times as great as the mass of one hydrogen atom.
(d) It contains seven electrons.
(e) It is able to form three covalent bonds.
(f) Its nucleus contains electrons.

Q3 Figure 6 of Modules 5/6 shows part of a protein molecule with atoms represented by spheres; Figure 18 of Modules 5/6 shows the same part of the same molecule but, this time, with atoms represented by their symbols (and with covalent bonds represented by lines).

(a) Use lines and symbols to show the structure of the following molecules: H_2, O_2, N_2, Cl_2, H_2O and CH_4.
(b) Name these substances and say whether they are solids, liquids or gases at room temperature.
(c) Which substances are chemical compounds and which are elements?

Q4 The chemical name for the alcohol found in beer and wine is ethanol. Using the same way of writing down molecular structures as in Q3 part (a), a molecule of ethanol has the structure shown in Figure 6 below.

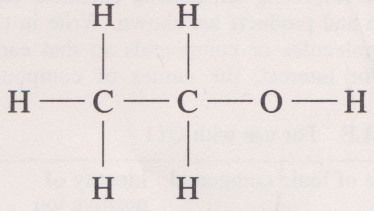


FIGURE 6 The structural formula of ethanol.

(a) What atoms are present in one molecule of ethanol? How many are there of each kind?
(b) The *structural formula* of methane is shown in the answer to Q3 (a); its *molecular* formula is CH_4. What is the molecular formula of ethanol?
(c) In ethanol, how many covalent bonds are formed by each kind of atom you identified in Q4 (a)?
(d) What is the relative mass of each kind of atom that you identified in Q4 (a)? You will need to refer to Table 6 in Modules 5/6.
(e) What is the relative mass of a molecule of ethanol?

Q5 Consult Table 6 in Modules 5/6 to find the information necessary to work out the ratio of the mass of one atom of nitrogen to that of one atom of silicon.

Q6 Propane, pronounced 'pro-pain', is a gas used in many gas burners. Its formula is C_3H_8. (a) What is its relative molecular mass? (b) Given that each of the carbon and hydrogen atoms form their normal number of covalent bonds, write down the only possible structural formula for propane.

Q7 In the following compounds, the atoms form the number of covalent bonds shown in Table 5 of Modules 5/6. Given this information, answer the following questions:

(a) What is the name and chemical formula of the covalent compound formed from carbon and sulphur?
(b) What is the name and chemical formula of the covalent compound formed from hydrogen and sulphur?
(c) Hydrogen bromide is a gas at room temperature that contains molecules of HBr. How many covalent bonds does bromine form?
(d) Using your answer to part (c), name and give the chemical formula of the covalent compound formed from carbon and bromine.

Q8 In one sentence or so, describe the difference between water (H_2O) and calcium oxide (CaO) in terms of their chemical bonds.

Q9 The following are the formulae of a number of ionic compounds. What are their names?

(a) NaCl (b) CaO (c) KI (d) $CaBr_2$ (e) ZnO
(f) MgO (g) Al_2O_3.

Q10 By referring to Table 8 in Modules 5/6 and the formulae of ionic compounds shown in Q9, work out the type and number of charges on ions formed from the following elements.

(a) aluminium
(b) iodine
(c) bromine
(d) zinc
(e) magnesium.

Show your answers by using the element's symbol together with superscripts of the type shown in Modules 5/6, Table 8. (When you have checked your answers, you could usefully add them to Table 8.)

Q11 Complete the Table at the bottom of this Section, using knowledge from Modules 5/6 and the answers to earlier questions. The top row is completed for you as are some parts of other rows.

Q12 In two or three sentences, say why ionic compounds do not exist as molecules.

Q13 In the following unbalanced chemical equations all the reactants and products are shown. Write in the numbers of atoms, molecules or compounds so that each equation balances. For interest, the names of compounds and/or elements are written below each. Note that As is the symbol for the element arsenic.

H_2	+	I_2	=	HI
hydrogen		iodine		hydrogen iodide
Co	+	O_2	=	CoO
cobalt		oxygen		cobalt oxide
As	+	Cl_2	=	$AsCl_3$
arsenic		chlorine		arsenic trichloride

Q14 In the following unbalanced chemical equations all the reactants and products are shown. Write in the numbers of molecules or compounds so that each equation balances. As before, the names of compounds and/or elements are written below each.

C_3H_8	+	O_2	=	CO_2	+	H_2O
propane		oxygen		carbon dioxide		water
MgO	+	HCl	=	$MgCl_2$	+	H_2O
magnesum oxide		hydrogen chloride		magnesium chloride		water
CH_4	+	Br_2	=	CBr_4	+	HBr
methane		bromine		carbon tetrabromide*		hydrogen bromide

(*also called tetrabromomethane!)

Q15 If the concentration of sodium nitrate in pond water is $1\ g\ l^{-1}$, how many grams are dissolved in $1\ cm^3$? (Recall that 1 litre = $1000\ cm^3$).

Q16 If 1.6 mg of sulphate ion is present in $256\ cm^3$ of a sample of mineral water, what is the concentration of this ion in $mg\ l^{-1}$?

Q17 Limestone consists mainly of the ionic compound calcium carbonate, $CaCO_3$.

(a) The positive ion in calcium carbonate is the calcium ion, Ca^{2+}. The negative ion is the carbonate ion; how would you write this?
(b) What is the ratio of the number of positive ions to negative ions in calcium carbonate?
(c) Calculate the relative mass of the carbonate ion given that the relative masses of carbon and oxygen atoms are respectively 12 and 16.

Q18 (a) If the relative mass of the phosphate ion is 95 (based on P = 31, O = 16, and the formula of the phosphate ion is PO_4^{3-}), calculate the mass of phosphorus in 95 g of phosphate ion.
(b) Water used in a particular industrial process must not contain more than 19 $mg\ l^{-1}$ of phosphate ion. How would you describe that limitation in terms of 'phosphate ion expressed as phosphorus'? (Use information from part (a)).

TABLE For use with Q11

Name of ionic compound	Identity of positive ion	Identity of negative ion	Ratio of no. of positive ions to no. of negative ions.	Formula of ionic compound
sodium chloride	Na^+	Cl^-	1 : 1	NaCl
calcium chloride	Ca^{2+}	...	1 : 2	...
sodium nitrate	...	...	...	$NaNO_3$
sodium sulphate	...	SO_4^{2-}	...	...
calcium nitrate	...	...	...	$Ca(NO_3)_2$
... ...	Al^{3+}	NO_3^-	...	...
aluminium sulphate	...	...	...	...

MODULE 7 QUESTIONS

Q1 Fill in the blanks making appropriate choices from the following list of twelve items. All items should be used once only; there is only one possible arrangement.

axes, dependent, horizontally, independent, origin, quantity, scale, title, units, variables, *x*, *y*.

(a) On a graph the ... variable is plotted vertically on the ... axis, and the ... variable is plotted ... on the ... axis.
(b) When plotting a graph, it is important to choose the ... for each axis in such a way that the ranges of experimental values for both ... conveniently fit on to the graph paper.
(c) When labelling each axis of a graph, the ... in which each ... is measured must be written on the axis. The graph must also have a ... .
(d) In many graphs the range of each variable, shown on each of the two ... , begins at 0. The single point where both axes register 0 is called the ... .

Q2 A, B and C are three entirely unconnected experiments. Conclusions from these experiments are:

In *Expt A:* Solubility of glucose increases with temperature up to a particular value. (Note: in this experiment, solubility is the maximum mass of solid that will dissolve in one litre of water.)

In *Expt B:* Over a period of time, the temperature of water in an insulated tank is constant.

In *Expt C*: The time a train takes to come to a halt increases as the speed of the train (at the moment the brakes are applied) increases.

The Table shows the variables in each experiment.

Expt.	Variable	D or I?	Units?	Graph no?
Expt A	solubility			
	temperature			
Expt B	time			
	temperature			
Expt C	train speed			
	stopping time			

(a) Decide, for each experiment, which of the listed variables is the dependent variable and which is the independent variable; enter your choices (D or I) in the first column of the Table.
(b) From the following list of units, select those that you think are appropriate for each measurement and enter them in the Table in the appropriate spaces. You will need to use some units more than once: °C, g l^{-1}, km hr^{-1}, min
(c) When measurements were complete, graphs were plotted from which the conclusions above were drawn. Match three of the five unlabelled sketch graphs shown in Figure 7 to each of Experiments A to C. Enter your choices in the appropriate spaces in the Table.

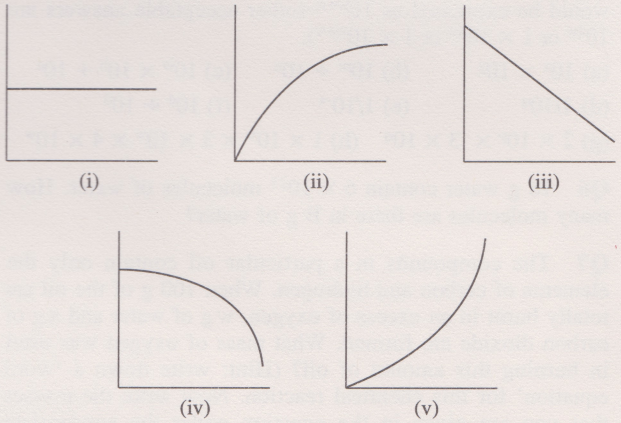


FIGURE 7 Sketch graphs.

Q3 Tabulated below are the average masses of voles at different ages. Without plotting the graph, look at the range of data presented in the Table and write down:

(a) What you would plot on the *x* and *y* axes of a graph intended to display the results.
(b) What scale you would use on each axis assuming you were restricted to the paper shown in Figure 8.
(c) How you would label each axis.
(d) What title you would give your graph.

Age/days	Mass/g	Age/days	Mass/g
0	2.1	28	15.8
1	2.2	34	16.9
2	2.4	40	17.4
10	4.0	46	17.7
16	7.7	52	17.9
22	11.6	58	17.9

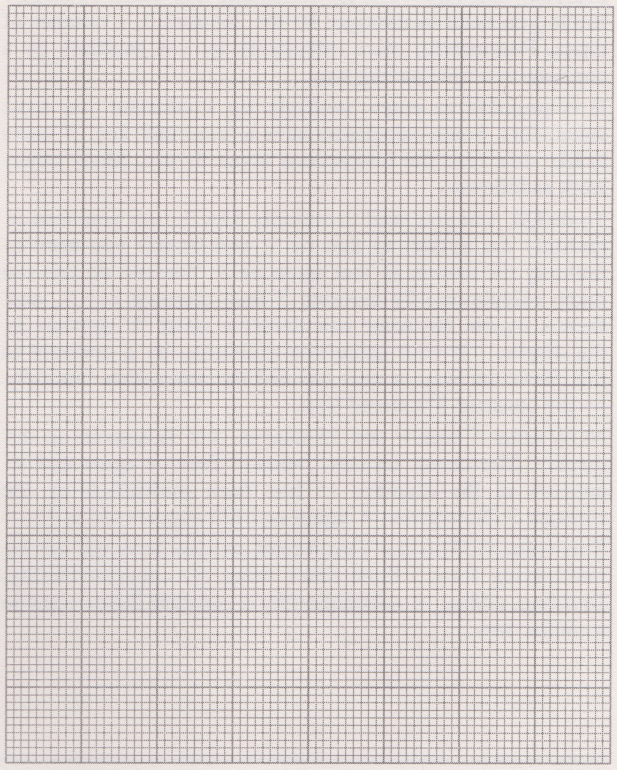


FIGURE 8 Graph paper for Q3 and Q4.

Q4 Use the data in the Table in Q3 to do the following.

(a) Plot the data on the graph paper in Figure 8. You should consider whether (and why) you should simply 'join up the points' or whether you should draw 'some other kind of line'. You should also consider whether or not *all* the points should actually be on the line. Check that you have labelled your axes and added the title.
(b) Write two or three lines describing in words the growth pattern depicted by the graph.

Q5 Each set of data, (a) to (d), represents measurements of mass made at equal time intervals. Although each set increases from left to right, only *one* of the four sets can be correctly described as showing *exponential growth.*

Revise the meaning of 'exponential growth' in of Module 7, Section 7, then—by doing whatever calculations you think are necessary—decide which one set of numbers illustrates exponential growth.

(a) 1, 3, 5, 7, 9, 11 (b) 4.2, 8.4, 12.6, 16.8, 21.0
(c) 1, 4, 9, 16, 25, 36 (d) 3.5, 10.5, 31.5, 94.5

Q6 Assuming each number is a measurement at equal time intervals, which one or ones of the following sets of numbers show(s) exponential change?

(a) 3, 5.9, 8.8, 11.7, 14.6 (b) 0.25, 1, 4, 16, 64, 256

(c) 100, 50, 25, 12.5, 6.25 (d) −2, −6, −18, −54, −162

Q7 Suppose the number of reported cases of a particular disease grows exponentially so that each year the number increases by a factor of 3. Calculate, without using a calculator, the number of cases each year for four years given that the number of cases in Year 1 was 5. Enter your answer in the grid below.

Year	1	2	3	4
No. cases	5	...	...	...

Q8 Repeat Q7, using your calculator in a way that has not been introduced before and which is described below.

In maths, the word 'constant' means what it does in everyday language—that is, 'something that doesn't change'. In Q8 you need to multiply by the constant 3, a number of times. Fortunately, most calculators let you multiply by a constant in a time-saving way.

It is possible to enter 'multiply by a constant' on most calculators. Some calculators have a 'constant' key. If yours is like this, read the instructions about its use. Many others work as follows:

To do Q8, *first* you need to multiply 5 by 3, *then* you need to multiply the answer to that by 3, and *finally* you need to multiply the answer to that again by 3. You can do this (on many calculators) by entering 3 then pressing key ×, and then × *again*. (In some calculators a small 'k' appears in the display.) If your calculator multiplies by a constant in this particular way, it now has had the instruction 'multiply by the constant 3' entered in it.

NOW enter 5 then = and 15 (that is, 5 × 3) appears in the display. The clever bit now follows! Press = again and 45 appears in the display; 15 has been multiplied by the constant 3. Now press = again and 135 appears (45 × 3).

This value (135) *is* the Year 4 total and you have got this by multiplying 5 by 3 three times using the constant facility. Dividing, adding or subtracting by a constant factor work in a similar way.

Q9 Can you work out a way to use the x^Y key (or y^X key) to calculate the Year 4 value in Q8? Experiment with your ideas: you know the answer should be 135!

Q10 When you have discovered how to do Q9 using the x^Y or y^X key (or have read the answer to Q9), calculate how many cases of the disease there would be in Year 15. (Ignore sf in answering this question.)

Q11 This exercise practises the idea that every point drawn on a piece of graph paper can be specified by two values—its x value and its y value.

The graph paper in Figure 9 has scales drawn for you. Plot the following points and join them up (one to the next) in a left to right direction:

(3, 4), (5, 1), (7, 4), (9, 7), (11, 10), (13, 13).

The picture you get should tell you whether or not you have done it correctly.

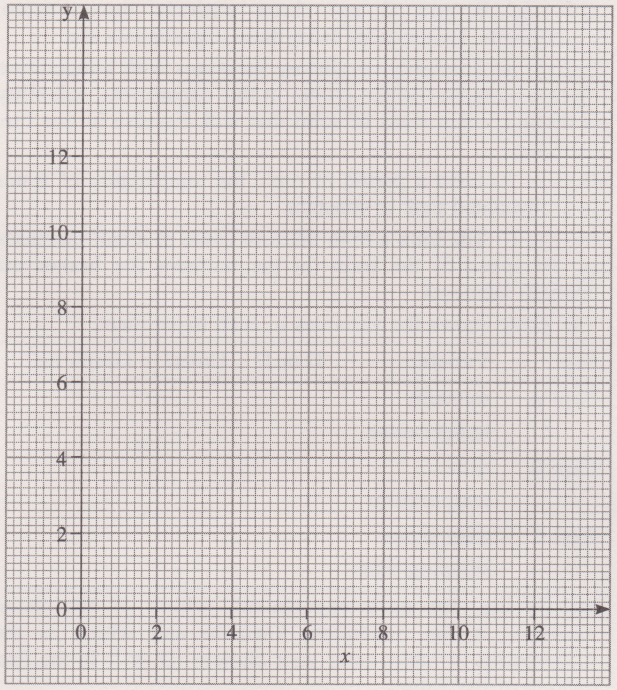


FIGURE 9 Graph paper for plotting points.

MODULE 8 QUESTIONS

Q1 The masses of six rabbits are 1.0 kg, 0.9 kg, 1.1 kg, x kg, y kg and z kg. What is the average mass (M_{av}) of these rabbits? (Hint: recall from Module 2 how averages are calculated. Your answer should take the form of an equation $M_{av} = \ldots$.)

Q2 A particular kind of wooden block is g cm long, h cm high and d cm deep (from front to back face). 50 of these blocks are built into a wall, arranged as 5 rows of 10 blocks. (Hint: it may help you to draw a diagram.)

(a) What is the volume of one block in cm^3?
(b) How high is the wall (in cm)?
(c) How long is the wall (in cm)?
(d) What area is the front face of the wall (in cm^2)?

Q3 The mass of a block of granite is M kg and its volume is V cm^3. Express the density (ρ) of the granite in kg m^{-3} in terms of M and V. (Recall that 10^6 cm^3 = 1 m^3.)

Q4 Sound from a pistol shot travels at v m s^{-1} from a boat directly to a vertical cliff where it is reflected directly back to an observer on the boat. He hears the echo t seconds after firing the pistol. The distance from ship to cliff is d km. Write an equation expressing d in terms of v and t.

Q5 Express the following in a way that involves *one* ten raised to the appropriate power. For example, $10^w \times 10^w$ would be expressed as 10^{w+w} (other acceptable answers are 10^{2w} or 1×10^{2w} or $1 \times 10^{w+w}$).

(a) $10^x \times 10^y$ (b) $10^m \div 10^n$ (c) $10^w \times 10^b \div 10^c$

(d) $1/10^q$ (e) $1/10^{-t}$ (f) $10^d \div 10^d$

(g) $2 \times 10^p \times 3 \times 10^q$ (h) $1 \times 10^w \times 3 \times 10^w \times 4 \times 10^w$

Q6 18 g water contain 6×10^{23} molecules of water. How many molecules are there in B g of water?

Q7 The compounds in a particular oil contain only the elements of carbon and hydrogen. When 100 g of the oil are totally burnt in an excess of oxygen, w g of water and x g of carbon dioxide are formed. What mass of oxygen was used in burning this amount of oil? (Hint: write down a 'word equation' for this chemical reaction. Now write the masses that you are given in the question under the appropriate parts of this equation. What DON'T you know? How can you work it out?)

Check your answers to Q1 to Q7 before continuing

Q8 Rearrange the following equations so that the quantity in brackets becomes the subject of the equation.

(a) $y = x + 9$ [x] (b) $g = h + 2x$ [h]

(c) $m = k + 2j$ [j] (d) $a = b + \sqrt{c}$ [c]

(e) $m = n + u^2$ [u] (f) $q = r/7$ [r]

(g) $d = 8/e$ [e] (h) $1/w + x = 9$ [w]

(i) $f = \frac{g}{5} - k$ [g] (j) $\frac{3}{v+2} = y$ [v]

(k) $y = mx + c$ [x] (l) $s = mvw$ [v]

Check your answers to Q8. Now do Q9 which is very similar in difficulty to Q8.

Q9 Rearrange the following equations to make the quantity in brackets the subject of the equation.

(a) $n = m/4$ [m] (b) $108 + x = y$ [x]

(c) $12 = r + 7q$ [q] (d) $s = t + 7u$ [t]

(e) $g = 6 + \sqrt{3h}$ [h] (f) $g = 6 + h\sqrt{3}$ [h]

(g) $a = b + 2c^2$ [c] (h) $x = \frac{4 + mn}{s}$ [m]

(i) $a = \frac{1}{\sqrt{b}}$ [b]

Q10 Solve the following equations. (This means, 'work out the numerical value of the unknown quantity represented by the letter'.) Solving them involves rearranging them. (f) to (g) are hard.

(a) $x^2 = 16$ (b) $\sqrt{y} = 7$ (c) $q - 2 = 10$

(d) $2m + 9 = 39$ (e) $4n^2 - 14 = 50$

(f) $\frac{4}{3}(\sqrt{r} + 1) = 12$ (g) $\frac{4}{3}\sqrt{r + 1} = 12$

Check your answers to Q9 and Q10 before continuing with problems involving words as well as algebra!

Q11 A meteorite with M kJ of kinetic energy hits the Earth's surface. On impact H kJ of heat energy and S kJ of sound energy are produced. Assuming this represents all the energy transformations at impact, write an equation expressing M in terms of H and S.

Q12 A block has dimensions a cm × b cm × 10 cm and a density of 2.4 g cm^{-3}.

(a) Using the formula:

kinetic energy = $\frac{1}{2}Mv^2$

calculate its kinetic energy when it is falling at 10 m s^{-1}.

(b) If a = 25 cm and b = 4.0 cm, calculate the kinetic energy in joules.

Note: when using the formula for kinetic energy, *M must* be in kg and and *v must* be in m s^{-1} to give an answer in joules.)

Q13 The larvae of blowflies (= maggots) always wriggle rapidly away from any source of light. The brighter the light, the faster they wriggle away. If the intensity of a light is *I* lumens, and its distance from the maggots is *d* cm, then the maggots' speed of retreat (v) in cm s^{-1} is given by

$$v = \frac{kI}{d^2}$$

Make *d* the subject of this equation. (It doesn't matter if this information is new to you, or if you have never met 'lumen' as the unit of light intensity. All you have to do is the algebra!)

Q14 Dirty river water contains A g l^{-1} nitrate ion, B g l^{-1} phosphate ion and C g l^{-1} of other dissolved solids.

(a) If Q litres of river water were evaporated to dryness (i.e. heated till all the water had been driven off), what total mass of solid would be left as a residue?

(b) What % by mass of this residue is nitrate ion?

Q15 A hydrocarbon is a chemical compound that contains nothing but hydrogen and carbon atoms.

(a) The hydrocarbon propane has the formula C_3H_8 and its combustion in oxygen is represented by the equation:

$C_3H_8 + 5O_2 = 3CO_2 + 4H_2O$

Check that this balances by counting the C atoms on the left and then on the right; do the same for the H atoms and the O atoms.

(b) Based on your understanding of this, work out the equation for the combustion in oxygen of the hydrocarbon C_xH_y (Assume that, as with propane, the products are only H_2O and CO_2. *This is included for fun: it is hard and may be omitted if you find chemical equations tricky.*)

Q16 There are 10 rabbits in an area of scrub-land in January 1980 and, through reproduction and colonization, their numbers increase by a constant factor of Q each year. (Note: this means that the number of them is multiplied by Q each year.)

(a) Is the growth in rabbit population exponential?

(b) How many rabbits should there be in January 1990?

Q17 What are the reciprocals of the following?

(a) ab (b) a/b (c) b/a (d) C^{-y} (e) s^{-1} (f) m^{-2}.

MODULE 9 QUESTIONS

Unless otherwise stated, take π = 3.142. Answers should be given to an appropriate number of significant figures. Remember to draw a diagram if you need one to help you visualise the statements in the question.

Q1 Calculate the circumference of circles given the following information about their radii or diameters. Give the answers in the same units as in each question and to an appropriate number of significant figures.

(a) radius = 4.6 cm (b) diameter = 104 metres

(c) radius = 1.496×10^{11} metres.

Q2 A plane flies round the equator of the Earth keeping exactly 7 km above sea level. A satellite circles the Earth directly over the equator at exactly 72 km above sea level. Take the Earth's diameter at the equator as 12 756 km and π = 3.141 6.

(a) Calculate the circumference of the circle (C_1) followed by the plane.

(b) Calculate the circumference of the circle (C_2) followed by the satellite.

(c) Calculate the ratio of C_2 to C_1.

Q3 Assuming the Earth to be perfectly spherical, what is the ratio of the length of the Greenwich meridian to the length of the equator? (Hint: no calculations are necessary.) Explain your answer in one or two sentences.

Q4 The diameter (D) of the Earth, measured in the plane of the equator, is 12 756 km. The distance around the equator at sea level (C) is 40 074 km.

(a) What is the ratio of C to D?

(b) Briefly comment on this numerical value.

Q5 Which of the following have constant lengths?
(a) Different lines of latitude.
(b) Different lines of longitude.
(c) Thickness of the crust measured at various places on the Earth's surface.
(d) Diameters of circles formed by different lines of longitude.
(e) Diameters of circles formed by different lines of latitude.

Q6 Without using your calculator, work out the following:
(a) $\dfrac{40}{14 \times \frac{22}{7}}$ (b) $\dfrac{25}{15 \times \frac{5}{6}}$
(c) using $\pi = \frac{22}{7}$, calculate the diameter of a circle of circumference 44 cm.

Q7 Calculate the areas of the following circles from the data given. Give the answers in the units used.
(a) radius = 4.6 cm (b) diameter = 144 m
(c) diameter = x cm.

Q8 Blood flows out from the human heart via arteries. In the body, these divide into arterioles—and the arterioles divide into capillaries. Assume that the cross section of each is a circle. The Table below shows the typical cross sectional diameter of each kind of blood vessel.
(a) Complete columns 2 and 3 of the Table.
(b) Can you see a way to calculate the ratio of the areas of each kind of vessel without using π?
(c) Calculate the ratio of the areas of each kind of vessel.

	(1) Diameter/ m	(2) Diameter/ mm	(3) Area/ mm^2
Artery	5.0×10^{-4}		
Arteriole	1.2×10^{-4}		
Capillary	2.5×10^{-5}		

Q9 Calculate the surface areas of the following spheres from the data given. Give the answers in the units used.
(a) radius = 4.6 cm (b) diameter = 102 m (c) radius = x cm.

Q10 What is the ratio of the surface areas of spheres of radii p cm and q mm?

Q11 The Sun produces 3.90×10^{26} J of energy every second. How many $J\,s^{-1}$ are radiated into space by each square metre of the Sun's surface area? (The Sun has a radius of 6.960×10^8 m.)

Q12 Calculate the volumes of the following spheres from the data given. Give the answers in the units used.
(a) radius = 4.6 cm (b) diameter = 102 m (c) surface area = 102 m^2

Q13 The radius of the Sun is 109 times as great as that of the Earth. Calculate the ratio of the volume of the Sun to that of the Earth. (Hint: you do *not* need to use the numerical value of π or the actual radii of Sun and Earth. Despite that, the question is fairly difficult.)

Q14 The radius of a hydrogen atom is 3.7×10^{-11} m.
(a) Calculate its volume in m^3 (to 2 sf). (b) Given that the volume of the Sun is 1.4×10^{27} m^3, calculate how many times greater the volume of the Sun is than the volume of a hydrogen atom (to 2 sf).

Q15 Calculate the size of angles a to i in Figure 10. (Hints: think about parallel lines, angles opposite to each other, the three angles in any triangle, angles on a line being 180°, and angles forming a circle being 360°. If you find the question hard, just start playing with ideas—marking the Figure with a pencil as necessary.)

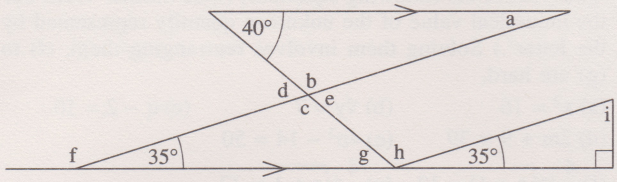


FIGURE 10 Some angles to calculate.

Q16 A ship sails by the shortest route from Aranuka in the Gilbert Islands (0°, 174° E) to the Galapagos Islands (0°, 91° W). Referring to Figure 28 in Module 9, answer the following:
(a) What is the name of the line of latitude on which the ship is sailing?
(b) In which direction is it sailing?
(c) What % of the total circumference of the Earth is the ship's journey?

These questions are about calculations; ignore the issue of significant figures in your answer unless otherwise indicated.

MODULE 10 QUESTIONS

Q1 Three brick walls form a triangle ABC with AB = 52 m, AC = 48 m and BC = 20 m. Using a scale of 1 cm = 10 m, draw triangle ABC to scale on the 1 cm graph paper in Figure 11.
(Draw AB horizontally near the bottom of the paper. Then, centre A, use compasses to draw an arc of an appropriate length to represent AC. Finally, centre B, use the compasses to draw an arc to represent BC. Where the arcs cross, mark point C. Join A, B and C with a ruler).

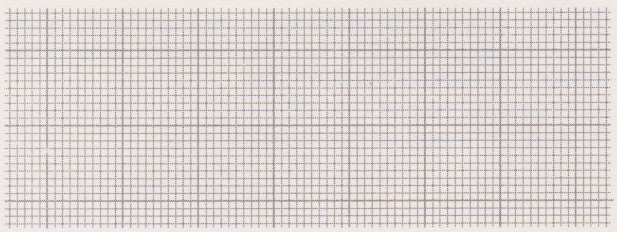


FIGURE 11 Graph paper for drawng a triangle.

(a) Using a protractor, measure and write down ∠ABC, ∠BCA and ∠CAB. (Note the use of the ∠ sign to mean 'angle' ; ∠ABC means 'angle ABC'.)
(b) Given your measured value for ∠BCA, how do you think the lengths of sides AC, CB and BA are related? Explain your answer. Use a calculator to check your idea.
(c) Add ∠ABC, ∠BCA and ∠CAB. What do you get? What did you expect?

Q2 Figure 12 shows a triangle XYZ. Measurement (which you can check if you wish) shows that ∠YZX = ∠BCA (of Q1) and ∠ZXY = ∠CAB (of Q1).

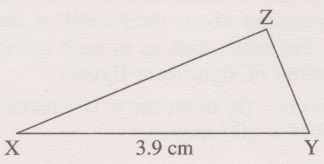


FIGURE 12 Triangle XYZ

When answering the following, use the actual measurements of triangle ABC shown in Figure 11.

(a) Without further measurement, how can you correctly deduce that ∠XYZ = ∠ABC (of Q1)?
(b) Triangles ABC and XYZ are both right–angled triangles. However, they are related in another way. Put one word into the blank space so that the sentence describes how ABC and XYZ are related.
As triangles ABC and XYZ have identical angles, they are said be ... triangles.
(c) You know the lengths (in cm) of AB, AC and BC from Q1. Measurement shows that XY = 3.9 cm. Calculate AB/XY. Express the ratio of AB : XY as whole numbers (which you should find).
(d) Using the relationship in (b), predict values for AC/XZ and CB/ZY. What is the ratio ZY/CB?
(e) Use these ratios to determine the lengths in cm of sides YZ and ZX and write these on Figure 12.
(f) As ∠YZX = 90°, how do you think the lengths of sides XY, YZ and ZX are related? Explain your answer. Use a calculator to check your idea.

Q3 Figure 13, (not drawn accurately) shows a beam of light travelling from torch T to a mirror which it hits at M. The light is then reflected along the path shown to a screen at S.

A scientist is interested in the relationship between the 'angle of incidence' (∠*i* in the Figure) and the 'angle of reflection' (∠*R* in the Figure). MN is at right angles to the mirror and, in this experiment, to the line TS. Measurement shows that ∠MTN = ∠MSN.

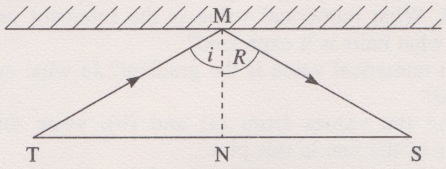


FIGURE 13 Relationship between ∠*i* and ∠*R*.

(a) What can you deduce about ∠*i* and ∠*R*? Explain your answer.
(b) What kind of triangle is TMS? On what grounds did you determine that?
(c) What, therefore, can you deduce about TM and SM?

Q4 The relationship between the angle of incidence and the angle of reflection (that you discovered in Q3) is true for all experimental arrangements of torch, mirror and screen.

Suppose the torch and screen are moved so that ∠*i* = 30°. Sketch the new triangle T′MS′ and comment on any special name or features that it has. (Note: The ′ to the right of the T and the S simply means 'new T' and 'new S'—because their position has changed and they are part of a new triangle).

Q5 This question makes further use of triangle XYZ in Figure 12. Check that you have correctly marked on that Figure that YZ = 1.5 cm and ZX = 3.6 cm and ∠YZX = 90°.
(a) Which side is the hypotenuse?
(b) For ∠X (shorthand for ∠YXZ), which side is termed *opposite* and which side is termed *adjacent*?
(c) For ∠Y, which side is *opposite* and which is *adjacent*?

Q6 Referring to Figure 12 and its annotations, complete the Table below. The first line of the Table has been completed for you.

Obtain the value of ∠X by using the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ buttons respectively (or, on some calculators, INV sin or SHIFT sin). As ∠X is the same angle throughout, your three entries in the third column should be the same.

Name of trig ratio	Ratio of sides	Ratio to 3 sf	∠X to 3 sf
sin X	$\frac{\text{opp}}{\text{hyp}} = \frac{ZY}{XY} = \frac{1.5}{3.9}$	0.385	22.6°
cos X	.....	.....	.....
tan X	.....	.....	.....

Q7 (a) By similar methods, calculate to 3 sf sin Y, cos Y and tan Y. Also find ∠Y and complete the Table below.
(b) Compare these values with those for sin X, cos X and ∠X. What relationships are there and how can these be explained?
(c) For interest, can you see any relationship between tan X and tan Y?

Name of trig ratio	Ratio of sides	Ratio to 3 sf	∠Y to 3 sf
sin Y	.....	.....	.....
cos Y	.....	.....	.....
tan Y	.....	.....	.....

Q8 In order to investigate friction, a brass disc is placed at the top of a sloping board down which it can slide. The disc just begins to slip when the angle between board and floor is 11.4°. At this point the top of the board is 9.00 cm vertically above the floor.
(a) Sketch this arrangement and use trigonometry to find how long the board is.
(b) What is the gradient of the board (expressed as a tangent)?
(c) Use this tangent to calculate the horizontal distance from the point of contact of the board with the floor to the point on the floor vertically under the top of the board.
(d) What mathematical relationship must there between your answers to (a) and (c) and the 9.00 cm length given in the Question? Check this on your calculator.

Q9 Gradients in science are normally expressed as the tangents of angles. However, on road signs gradients are described differently. Old–style signs are expressed as '1 in n' which means 1 metre up for every n metres horizontally. On newer signs the gradient is given as a percentage—*x*% means *x* metres up for every 100 metres horizontally.

Fill in the blanks in the Table below by converting the given gradients appropriately. The top row has been done for you. Work to three sf where relevant.

angle of slope(α)	gradient (tan α)	% (*x* in 100)	1 in n
0.57°	0.01	1.00	1 in 100
1.43°	...	...	...
...	0.0833	...	...
...	0.100	...	...
...	...	...	1 in 7
...	...	...	1 in 3
...	...	50.0	...
...	...	100	...

Q10 The Leyland cypress is a rapidly growing evergreen tree. The Table below shows the height of a particular specimen over a period of 20 years from the time of planting ($t = 0$). Also shown at each value of t is the *change in height* (Δh) since the time of planting. Note the Greek letter Δ is pronounced 'delta' and means 'change in'.

t/yr	h/m	Δh/m
0	1.50	0
1	2.60	1.10
2	3.70	2.20
3	4.80	3.30
6	8.10	6.60
10	12.6	11.0
15	18.0	16.5
20	23.5	22.0

Plot h against t on Figure 14, and Δh against t on Figure 15. Now answer (a) to (f) of Q10 and (a) to (e) of Q11.

Height against time

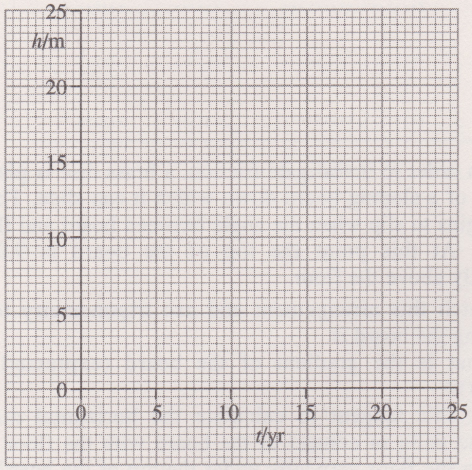


FIGURE 14 Graph to plot height against time.

Change in height against time

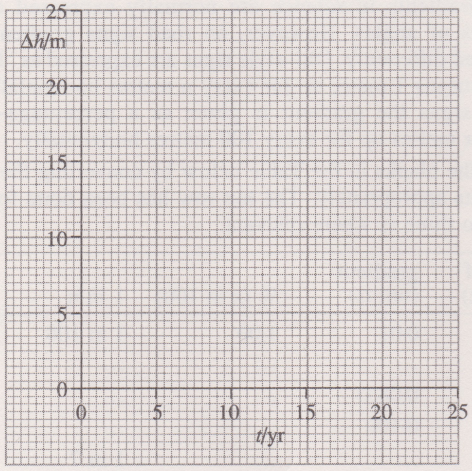


FIGURE 15 Graph to plot change in height against time

(a) Find the gradients of each line, expressing them in appropriate units. What do you notice about these values?

(b) In Figure 14, what does the intercept on the h axis (i.e. the y axis) represent?

(c) Which *one* of these mathematical statements is true: $h \propto t$ or $\Delta h \propto t$?

(d) Express the statement (chosen in (c)) with *words* instead of h or Δh, t and the $\propto$ sign.

(e) Convert the statement (chosen in (c)) into an equation using a constant of proportionality, k.

(f) What is the numerical value of that constant? How does it relate to your answers in (a).

Q11 The equation of a straight line is $y = mx + c$

(a) In any straight line graph, what do y, x, m and c represent?

(b) What variables are represented by y and x in Figure 14.

(c) What variables are represented by y and x in Figure 15?

(d) Using some of your answers to Q10, what are the numerical values of m and c in Figure 14? Use these to write the equation for the line plotted in this Figure.

(e) What are the numerical values of m and c in Figure 15? Use these to write the equation for the line plotted in this Figure.

Q12 The carbon dioxide concentration (C) in the air of a large sealed cupboard is 350 ppm (ppm = parts per million). A jar of fermenting grape juice is placed inside the cupboard and, as a consequence, the CO_2 concentration rises by 50 ppm each hour—starting from the time ($t = 0$) that the jar was introduced.

(a) When C/ppm is plotted against t/hours a straight line is obtained. What numerical value is the intercept on the C axis? In what units is it expressed?

(b) What numerical value is the gradient? In what units is it expressed?

(c) Using the values from (a) and (b), write down the equations of the line in this plot.

(d) Use that equation to predict the CO_2 concentration nine hours after introducing the fermenting juice.

Check your answers to Q12 before doing Q13.

Q13 A certain concentration of chlorine (in ppm) is present in the air above an enclosed swimming pool. The chlorination equipment develops a leak at $t = 0$ and the concentration of chlorine in the air rapidly increases.

Concentration is measured throughout the first hour, and a plot of *chlorine concentration* against *time* is found to be a straight line. In general terms, therefore, the equation for this line is given by $y = mx + c$.

After 26 minutes the concentration is 550 ppm and after 31 minutes it is 650 ppm.

(a) From the change in concentration between 26 minutes and 31 minutes, deduce the rate of concentration increase. What are its units?
Which letter represents this numerical value in the equation $y = mx + c$?

(b) At 26 minutes after the leak started, what are the numerical values of y and x in the equation?

(c) Use information from (a) and your answers to part (b) to calculate c in the equation $y = mx + c$. Thus, state the concentration of chlorine in the air before the leak began.

(d) Write down the equation for this particular line, making use of your calculated values for m and c.

(e) Use the equation to calculate the concentration of chlorine 10 minutes after the system started leaking.

MODULE 1 ANSWERS AND COMMENTS

Q1 (a) 1 133 (b) 1 059 (c) 6 005 (d) 456 (e) 37

• Note the spaces in the answers to (a) to (c): see Module 1, Section 6.2 for this convention. In hand written answers it is still common (and acceptable) to use commas e.g. 1,133.

• Note that when doing a series of additions as in (b) you do not need to press the = key until the end. Key in 5 then + then 791 then + then 263 then =. Note that the calculator display shows the result of the first addition immediately after you've entered the second +.

• The same idea of 'saving the equals sign to the end' applies to any sequence of operations e.g. the series of additions and subtractions in (e).

Q2 (a) 11 001 (eleven thousand and one) (b) 976 543 (nine hundred and seventy six thousand five hundred and forty three) (c) 1 000 000 (one million) (d) 100 000 120 (one hundred million one hundred and twenty) (e) 2 000 000 000 (two billion or two thousand million.)

• Writing very long numbers like this is cumbersome. In Modules 5 and 6 you will come across the number 600 000 000 000 000 000 000 000! In fact a different way of writing large numbers is usually used in science and you learn about this in Module 4.

• Note the answer to (e). Now in the UK—as in the USA and in the rest of the EC—a billion is one thousand million: a 'one' followed by nine 'noughts', viz. 1 000 000 000.

Q3 (a) 78 (b) 780 (c) 10

• To calculate (a), enter 1 + 2 + 3 + 4 ... + 12 = 78. Obtain (b) by subtracting 78 from her original 858 grains. If she has 780 grains left and each tray uses up 78 grains, she can fill 780 ÷ 78 more trays.

Q4 (a) 80 (b) 80 (c) 336 (d) 36 (e) 42 (f) 21

• You may be able to do many of these without the calculator! However, to get them right you would have to remember the BODMAS rules and apply them correctly. The great thing about doing them on your calculator is that the rules are usually applied for you with total accuracy—so long as you press the correct keys.

• Make sure you can do (c), (e) and (f) on the calculator: some people try to avoid the 'brackets' keys.

• Note that the answers to (a) and (b) are the same. In maths it does not matter in which order you multiply numbers together.

• If you got (f) wrong, did you remember to enter the × omitted by convention in the question: 3 × (6 + 1)?

Q5 (a) 28 (b) 1 780 (c) 6 (d) 33 (e) 33 (f) 9 (g) 36 (h) 36

• Note that the answers to (d) and (e) are the same: the brackets in (e) are optional in that they are just added for clarity. Similarly, the brackets in (g) aren't absolutely necessary (because BODMAS says divide before multiply). However, they certainly make (g) look clearer!

Q6 (a) 9 (b) 36

• Part (a) is exactly the same as part (f) of Q5. In complicated looking fractions, you must always 'completely work out the top', then 'completely work out bottom', then divide the top by the bottom. There are some short–cut methods that you learn about later. Part (b) is exactly the same as parts (g) and (h) of Q5.

Q7 (a) 900 g (b) 6 300 g (c) one third.

• Total daily food is mass eaten by mice plus mass eaten by rats. The mice eat 12 lots of (5 + 15 + 5) g and the rats eat 20 lots of (7 + 20 + 3) g. Remember 'lots of' means ×.

• Answer (b) = 7 × the part (a) answer.

• Answer (c) = amount mice eat (300 g) divided by total amount eaten (900 g).

Q8 (a) 10.04 (b) 0.09 (c) 0.91 (d) 9.12 (e) 2.24 (f) 1.67 (g) 61.835 (h) 1.286 (i) 0.888 (j) 1.333

• Look at (b) to (d) carefully and see what effect multiplying by 0.1, 10 and 100 has on the position of the decimal point. Remember in (b) not to waste time entering the 0 of 0.9.

• In (e) the calculator display answer has a 2 in the third decimal place (2.242 8...). When corrected to two decimal places this gives 2.24. Similarly, in (f) 1.666 corrects to 1.67. Appropriate corrections are made in (g) – (j).

Q9 (a) 360 litres (b) 5.4 litres (c) 10.8 times

• With 12 breaths of 0.5 litres each in one minute, he breathes in (and out!) 60 × 12 × 0.5 litres in one hour.

• From his normal 'breathed in' position, he can force himself to breathe in 2.1 litres more. From his normal 'breathed out' position, he can force himself to breathe out 1.3 litres more. Between the 'normal in' and 'normal out' positions there is 0.5 litres. Thus the total volume between 'fullest in'and 'fullest out' is:

2.1 litres + 0.5 litres + 1.3 litres = 3.9 litres. But his residual lung volume = 1.5 litres, therefore total volume of John's lungs is 3.9 litres + 1.5 litres = 5.4 litres.

• The total volume John's lungs hold is 5.4 litres. The normal 'in' is 0.5 litres. Therefore maximum is 5.4/0.5 times bigger than the normal volume. This is 10.8 times as great.

Q10 (a) 20.9 g (b) 83.6 g

• 25.0 g fresh grass became 4.1 g dried grass. Therefore water lost on drying was 25.0 g – 4.1 g = 20.9 g.

• If there are 20.9 g water in 25.0 g fresh grass, there are 20.9/25.0 g water in 1 g fresh grass. Therefore there are 20.9/25.0 × 100 g of water in 100 g fresh grass. This equals 83.6 g water. This is the percentage of water (parts in 100). You learn more about percentage in Module 2.

Q11 (a) 0.083 (b) 0.133 (c) 0.011

• To do this, divide the top number (numerator) by the bottom number (denominator) e.g. divide 1 by 12. You need to correct to three places of decimals in parts (a) and (b).

Q12 (a) –10 (b) 20 (c) 34 (d) –145 (e) 80 (f) –1 284 (g) –14 (h) 142 (i) –183.64 (j) 4.87 (k) 2.34

• These follow directly from the method of calculation described in Module 1, Section 8.

Q13 14 °C

• Do this as follows:

laboratory temperature is (– 31 °C) + (45 °C) = 14 °C.

MODULE 2 ANSWERS AND COMMENTS

Q1 (a) 42 600 m (b) 1.3 m (c) 0.970 m
(d) 0.047 58 m.

• In (d), notice the four digits to the right of the decimal point are grouped in threes from the point with a space after each group of three.

Q2 63.072 km (or 6.31 km to 3 sig figs).

• In 365 days there are 365 × 24 × 60 minutes. In each minute the tip travels 120 mm. So in 365 days it travels 365 × 24 × 60 × 120 mm = 63 072 000 mm. Dividing by 1 000 converts that to m. Dividing by another 1000 converts to km. So in a year the tip travels:

$\frac{63\,072\,000}{1\,000 \times 1\,000}$ km.

• When you do this kind of problem, 'overdo' rather than 'underdo' the number of steps. Your thinking process may well involve 'talking to yourself'. In fact, in problems that are harder, you might find talking out loud helps.

• Millimetres are not an appropriate unit–not when there are more than 63 million of them! Dividing by just 1 000 won't help much: 63 thousand metres is still a large number. So, divide again by 1000 to get 63.072 kilometres. On your calculator, either divide 63 072 000 by 1 000 000 or divide by 1 000 and by 1 000 again. It's the same thing. Quantities in the question are to three significant figures—therefore the answer is also to three significant figures.

Q3 (a) 816 mm (b) 0.85 mm in one hour.

• 0.816 m × 1 000 = 816 mm.

• It grows 816 mm in 40 days:

$\therefore \frac{816}{40}$ mm in one day, thus $\frac{816}{40 \times 24}$ mm in 1 hour.

By calculator, *either* divide 816 by 40 then divide the result by 24 *or* divide 816 by (40 × 24). Note that dividing by 40 and again by 24 is quicker. Remember from the Study Guide that the sign '∴' means 'therefore'. You should pronounce it to yourself in this way when reading maths involving it. Treat 40 days and 24 hours as exact times.

Q4 1 010 kcal per day.

• Add the five values (= 5 050 kcal) then ÷ by 5 days.

Q5 16 metres per minute.

• She crossed the lake 14 × 2 times. It is 40 m wide ∴ she swims 14 × 2 × 40 m. As she did this in 70 minutes, her average speed was:

$\frac{14 \times 2 \times 40}{70}$ = 16 m per minute.

Note that, as she may have swum at different speeds during the many crossings, 16 m per minute is the average speed.

Q6 2.30 g.

Average mass of eight insects = 1.75 g

Total mass of eight = 1.75 × 8 = 14.00 g

Mass of seven given = 11.70 g

Mass of the eighth = 14.00 g – 11.70 g = 2.30 g to 2 dec.pl.

• You can see without calculating that the eighth stick insect will have a mass substantially greater than the average (of 1.75 g). This is because all the other seven have masses equal to or less than the average!

Q7 (a) 3 (b) 0 (c) 1 (d) 4 (e) 1

• Simply count the digits to the right of the decimal point. See Module 1, Section 7.

Q8 (a) 5 (b) 1 (c) 2 (d) 5 (e) 1

• Count the digits wherever they are (omitting any non–significant zeros). In (c) the 0 counts as a significant figure as 3.0 means 'nearer to 3.0 than 2.9 or 3.1'. In (e) the 0 does not count as a significant figure; all we know is that the quantity is nearer 0.1 than 0 or 0.2, see Module 2, Section 4.1.

Q9 (a) 1.75 (b) 2 100 (c) 1.06 (d) 475 (e) 21.5

• (c) is rounded up because of the last 5 in 1.055 (d) is nearer to 475 than 476 (e) again, it rounds up.

Q10 2 hrs 53 min.

• $\frac{248 \text{ km}}{86 \text{ km hr}^{-1}}$ = 2.883 7 hr

Note: We have to turn the 0.883 7 hr into minutes by multiplying this part by 60. This gives 53 minutes (to the nearest minute).

Q11 (a) $\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) $\frac{5}{13}$ (e) $\frac{1}{3}$

• See Module 2, Section 5.1. (d) is already in its lowest terms. In (a) you needed to spot that 3 will divide into the top of the fraction (numerator) and also into the bottom of the fraction (denominator). The result of dividing both top and bottom by 3 is '3 over 4' that is, 3/4. In (e) it is harder unless you spot straight away that numerator and denominator are both divisible by 16. However, the answer may be approached in stages if necessary. To do this you divide top and bottom by *any* small number that will 'go into' both. And then do that again and again until you can't divide any more.

Q12 (a) 8 : 9 (b) 9 : 8 (c) 29 : 5 (d) 5 : 29
(e) 70 : 75 which is 14 : 15 (f) 2 : 3 (g) 7 : 17
(h) 53% (i) 47%

• From the data in Q12, there are 80 males, 90 females, 145 browns, 25 whites and 170 butterflies altogether. From this, whole number ratios (in lowest terms) can be determined. For example, 80 : 90 simplifies to 8 : 9 (pronounced 'a ratio of eight to nine'). Note that (b), ratio 9 : 8, is simply the reverse of (a).

• Note that all these ratios *can* be expressed as '1 : something' or 'something : 1', e.g., 29 : 5 is 1 : 5/29 which is 1 : 0.17 (2 sf). Sometimes the '1 to' bit is assumed and a ratio of 0.17 is given. This question asks for whole numbers.

• The percentages are best calculated from first principles (that is, remembering a 'per cent' is 'how many out of one hundred'). So, in (h), we have 90 females out of 170 altogether. This is:

$\frac{90 \times 100}{170}$ out of 100, namely 53% (to 2 sf).

The answers to (h) and (i) should add up to 100.

Q13 438 petrol cars.

• As there are 7 petrol cars to every 1 diesel car, in a total of 8 cars there will be 7 petrol cars. The car park contains 500 cars. How many 'lots of 8' in 500? Clearly, 500/8! So there will be:

$7 \times \frac{500}{8}$ petrol cars.

This is 437.5 which is 438 to the nearest whole number.

Q14 (a) 200 mm (b) 5 : 2

• The width is $\frac{40}{100}$ of length. So length is $\frac{100}{40}$ × width.

In (b) the unsimplified ratio is 200 : 80. This becomes 5 : 2. As the question did not specify a ratio expressed in whole numbers, 2.5 : 1 (or just 2.5) are also acceptable answers.

MODULE 3 ANSWERS AND COMMENTS

Q1 (a) 1 220 cm (b) 49.1 cm (c) 0.14 m (d) 950 mm (e) 197 cm (f) 1.10 m.

Q2 (a) 60 mm^2 (b) 38 mm^2 (c) 20 m^2.

Q3 (a) 0.1 m^2 (b) 1 000 cm^2 (c) 280 cm^2 (d) 28 000 mm^2.

• Note from the words of the question itself that areas (a) and (b) are equal. Thus 0.1 m^2 *equals* 1 000 cm^2. Similarly, areas (c) and (d) are equal, so 280 cm^2 equals 28 000 mm^2. You need this information for Q5 below.

Q4 (a) 37.5 m^2
(b) 37 500 000 mm^2
(c) In this question it is more sensible to use m^2 than mm^2 as huge numbers are avoided.

• Note from the words of the question that the areas refer to the same garden! Thus 37.5 m^2 equals 37 500 000 mm^2. You need this information for Q5 below.

Q5 Your completed table should look like this:

1 m^2	=	10 000 cm^2
1 cm^2	=	100 mm^2
1 m^2	=	1 000 000 mm^2

• Q5 asked you to work these 'conversion factors' out by making use of the answers to Q3 and Q4.

• Look back at the comment in the answer to Q3. You know from that question that 0.1 m^2 equals 1 000 cm^2 so it follows that 1 m^2 equals 10 000 cm^2. Similarly, 280 cm^2 equals 28 000 mm^2, so 1 cm^2 equals 100 mm^2.

• Look back at the comment in the answer to Q4. You know from that question that 37.5 m^2 equals 37 500 000 mm^2, therefore it follows that 1 m^2 equals 1 000 000 mm^2.

• It is *easier* to work out the relationships by picturing diagrams of the type in Figure 2. Here 1 cm^2 clearly contains 100 small squares, of 1 mm^2.

Q6 (a) 20 000 cm^2 (b) 3 000 000 mm^2 (c) 9 cm^2 (d) 16 m^2 (e) 25 m^2.

Q7 Approximately 7.2 m.

• Room area × 2.5 ≈ lung area of 90 m^2 ∴ room area ≈ 90 m^2/2.5. This is 36 m^2. So, 5 m (width) × length ≈ 36 m^2. Thus, length = 36 m^2/5 m ≈ 7.2 m. 'Thus' is often used in maths and means 'it follows that'.

• Note that although 90 ÷ 2.5 is exactly 36 and although 36 ÷ 5 is exactly 7.2, the answer can only be 'approximate' because the teacher said '*around* 90 m^2' and 'about two and a half times'.

Q8 7.92 m.

• Length = (13.08 × 100) cm. Each tile is 1.20 cm wide. So, a row stretching the full width of the room will contain (13.08 × 100 ÷ 1.2) tiles. This is 1 090 tiles (see Figure 16). As there are 719 400 tiles altogether, there must be 719 400 ÷ 1 090 = 660 rows. The other dimension of a tile is also 1.2 cm (they are square). So the room width is 660 × 1.2 cm = 792 cm = 7.92 m.

Q9 (a) 1 320 mm^3 (b) 1.32 cm^3 (c) 90 m^3 (d) 90 000 000 cm^3.

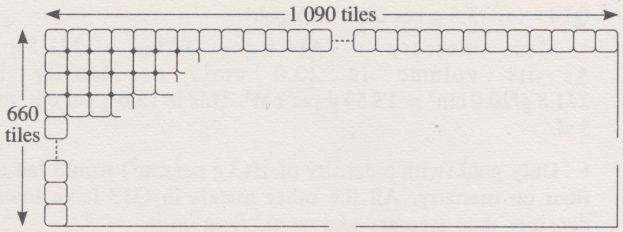


FIGURE 16 A rectangular tiled floor, for use with Q8.

Q10 (a) 1 000 mm^3 = 1 cm^3
(b) 1 000 000 cm^3 = 1 m^3
(c) 1 000 000 000 mm^3 = 1 m^3
(d) one billion mm^3 = 1 m^3.

• (a) and (b) of Q9 describe the same volume. So, from the answers to (a) and (b) of Q9, 1 000 mm^3 = 1 cm^3.

• (c) and (d) of Q9 describe the same volume. So, from the answers to (c) and (d) of Q9, 1 000 000 cm^3 = 1 m^3.

• From these two conversion factors (1 000 mm^3 = 1 cm^3 and 1 000 000 cm^3 = 1m^3), it is clear that to convert cubic metres to cubic millimetres it is necessary to multiply by 1 000 × 1 000 000. (This equals 1 000 000 000, also called *one billion*.)

Q11 (a) 1 400 cm^3 (b) 0.63 m^3.

• The answer to (a) is:
200 cm × 8 cm × 0.90 cm = 1 440 cm^3. To 2 sf this is: 1 400 cm^3. Similarly, (b) gives 0.6264 m^3 which is 0.63 m^3 when corrected to 2 sf.

Q12 4.0 cm in the first minute.

• Look back at Figure 3 in the Question. In the first minute the level in tank A falls by 12 cm. It follows that a volume of 12 cm × 60 cm × 100 cm, equalling 72 000 cm^3, flows out in one minute. This flows into tank B. (As this second tank has a *bigger* floor area than A, common sense tells us that the depth will not increase by as much as it has fallen in A.)

• When the 72 000 cm^3 water enters B, plainly it must occupy the same volume! So, 72 000 cm^3 must equal the increase in depth of B × 120 cm × 150 cm.

From this, increase in depth = $\frac{72\,000\ cm^3}{(120\ cm \times 150\ cm)}$

This gives 4.0 cm.

• Note for interest, that the amount of water flowing from tank A to tank B in each *subsequent* minute will become less and less (as the levels become more and more nearly equal).

Q13 Values are (a) 2.170 (b) 19.30 (c) 8.930 (d) 11.34. Gold has the greatest density of the metals listed.

• It is unnecessary (and, by convention, wrong) to repeat *units* within any Table if they are given at the head of a column or beginning of a row: see Module 2, Section 3.2.

• Each entry was calculated by dividing the mass of the cube in grams by its volume of 8.000 cm^3.

• Because each cube has the same volume, the cube with the greatest mass must have the greatest density. So, gold can be picked out as having the greatest density.

Q14 (a) 13.6 g per cm^3 (b) gold.

• Mass mercury = 414.8 g − 143 g = 271.8 g. As its volume is 20.0 cm^3, its density is 271.8 g/20.0 cm^3 = 13.59 g per cm^3. This is 13.6 g per cm^3 to 3 sf.

• Only gold (with a density of 19.3 g per cm^3) would fail to float on mercury. All the other metals in Q13 have lower densities than mercury and would float on it.

Q15 (a) 1 900 kg per m^3 (b) 900 kg per m^3 (c) 1.3 kg per m^3 (d) 0.18 kg per m^3.

• Note that densities in g per cm^3 are converted to densities in kg per m^3 by multiplying by 1 000. You can check this by doing one 'from first principles'. 1 cm^3 of bone has a mass of 1.9 g. So 1 m^3 of bone has a mass of 1.9 × 1 000 000 g. Dividing the result of that by 1 000 (to convert g to kg) we get 1.9 × 1 000 kg per m^3.

• You may not have thought about gas densities before. It is interesting to picture a tank 1 m × 1 m × 1 m and to realise that there is about 1 kg of air in it!

• A balloon filled with helium (even after taking the mass of the fabric of the balloon into account) has a *lower density than air*. So it would float.

Q16 (a) and (f).

• Recall from the Study Guide that, just as the sign '= ' means 'equals', the sign '≠' means 'NOT equal to'.

• Where Pythagoras' theorem is obeyed, the triangle must be a right–angled triangle. So, try this out on each:
(a) $1^2 + 2^2 = 5 \neq 3^2$ ∴ it is NOT a right–angled triangle
(b) $3^2 + 4^2 = 25 = 5^2$ ∴ it is a right–angled triangle
(c) $4^2 + (7.5)^2 = 72.25 = (8.5)^2$ ∴ it is a right–angled triangle, and so on.

Q17 30 cm^2.

• Horizontal × perpendicular = 60 cm^2. Half of this is 30 cm^2.

• The horizontal side is often called the 'base' and the perpendicular is often called 'height'. You may have come across the rule for the area of a triangle in some previous course as:

$\frac{1}{2}$ (base × height). Use this wording if you prefer it.

Q18 141 flags.

• The length of the hypotenuse (hyp) is calculated by Pythagoras' theorem. $Hyp^2 = 10^2 + 10^2 = 200$. So horizontal distance along the top of each flag is $\sqrt{200}$. This = 14.14 cm. To find how many flags fit along the taut string 20 m (= 2 000 cm) long, divide 2 000 by 14.14. This is 141.44 flags. In real terms, we work in whole flags, hence the answer is 141 flags.

MODULE 4 ANSWERS AND COMMENTS

Q1 (a) 8×10^2 (b) 8.00×10^2 (c) $1.239\,0 \times 10^3$ (d) $1.230\,0 \times 10^3$.

• In (a), 800 is expressed to 1 sf only—hence we write 8×10^2. Note that the *single* digit '8' to the left of × indicates *one* significant figure. In (b), 800 can be shown unambiguously to 3 sf in scientific notation *in that three digits* (8.00) are shown to the left of the × sign.

• Note the general point that scientific notation always permits the writer to be precise about significant figures without the need for any other comment. For example, £800 could mean *exactly* £800: it would to a bank! Or, it could mean nearer £800 than either £700 or £900—e.g. the approximate cost of a holiday. If you wished to indicate which, you should say 'to 2 sf' or 'to 3 sf'. Using scientific notation, you simply write it as shown in (a) and (b) above.

• In (c), 1 239.0 is plainly to 5 sf (count the digits). The convention explained above accounts for the answer shown.

Q2 (a) 1 400 (2 sf) (b) 1 400 (4 sf) (c) 249.31 (5 sf).

• (c) is plainly to 5 sf. Note that the information in the brackets is essential in (a) and (b) because 1 400 can mean nearer to 1 400 than 1 500 (2 sf) or it can mean exactly 1 400 (4 sf).

Q3 (a) 3.6×10^8 km^2 (b) 1.5×10^{11} m.

• In (a), replacing the word 'million' gives 360×10^6 km^2. In scientific notation this is 3.60×10^8 (360 has been divided by 100 and 10^6 has been multiplied by 100). In (b) the word 'billion' has been replaced by 10^9 and then converted to scientific notation.

Q4 (a) 6×10^{-2} (b) 8.0×10^{-2} (c) 2.40×10^{-5} (d) 1.00×10^1 (also 1.00×10) (e) 1.01×10^0.

• In (a), to convert 0.06 to 6 *multiply* by 100. To keep the overall number the same, now *divide* by 100 which is the same as multiplying by 10^{-2}. Similarly, to get 2.40 in (c) *multiply* by 100 000 and to keep the overall number the same now *divide* by 100 000 which is the same as multiplying by 10^{-5}.

• In (d), it is usual to omit the '1' in 10^1. In (e), 10^0 is the same as 1.

Q5 (a) 1.836×10^6 (b) 1.22×10^7 (c) 1.94×10^{-3} (d) 2×10^{-14}.

Q6 (a) 1.16×10^6 (b) 5.98×10^9 (c) 9×10^3 (d) 1.997×10^2 (e) 5.22×10^{-2} (f) 2.1×10^4.

• Numbers that are expressed using powers of 10 can be subtracted or added *provided* they involve the SAME powers of 10. You can choose for yourself 'which power of 10' to work in. As you're asked to express the answers in scientific notation, some choices are better (involve less work) than others.

• (a) becomes $1.3 \times 10^6 - 0.14 \times 10^6$. This is simply $(1.3 - 0.14) \times 10^6$ (b) becomes $6 \times 10^9 - 0.02 \times 10^9$. This is simply $(6 - 0.02) \times 10^9$. In (c), remember that 10^4 is really 1×10^4 This can be rewritten as 10×10^3. Thus we get $(10 - 1) \times 10^3$. In (d), we have to convert 3×10^{-1} into 'something' $\times 10^2$ so that it can be subtracted from 2×10^2. This becomes $2 \times 10^2 - 0.003 \times 10^2$.

• In (e) and (f), addition follows the same rules. Once again, note that 10^3 really means 1×10^3. This becomes, therefore, 0.1×10^4. This can then be added to 2×10^4 to give the answer of 2.1×10^4.

Q7 The answers are as in Q6.

- You do Q6 (a) as follows: Enter 1.3, press EXP (or EE), 6, –, 1.4, then press EXP (or EE) again, then 5 and finally =. The calculator will probably display the answer in ordinary (not scientific) notation—in this case 1 160 000. As usual, to turn this into scientific notation, count the 'number of places to the left' that you have to move the decimal point to give 1.16. In fact you move it six places. This means that you now write down $\times 10^6$, thus giving 1.16×10^6.

Q8 $3.004\,5 \times 10^{13}$ blood cells.

- Simply add $0.004\,5 \times 10^{13}$ to 3×10^{13}.

Q9 (a) 10^{12} (b) 10^{-5} (c) 1 (or 10^0) (d) 10^1 or just 10 (e) 1.65×10^{23}.

- All these are multiplications of power of ten. To do these, add the powers (paying attention to their signs).

- In (c), remember 10^0 equals 1. In (e), as well as adding the indices, the ordinary numbers have to be multiplied: $3 \times 5.5 = 16.5$ so $3 \times 10^{13} \times 5.5 \times 10^9 = 16.5 \times 10^{22}$. Putting this into scientific notation, we have 1.65×10^{23}. (Note for interest: as 5.5×10^9 is the number of people on the Earth and 3×10^{13} is the number of red blood cells per person, you have calculated how many human red blood cells there are on the planet!)

Q10 (a) 1 (or 10^0 or 1×10^0) (b) 10^5 (or 1×10^5) (c) 2×10^2 (d) 10^6 (or 1×10^6) (e) 10^{12} (or 1×10^{12}) (f) 15 (or 1.5×10^1) (g) 10^{15} (or 1×10^{15}) (h) 10^9.

- As before, multiplication involves adding the powers and division involves subtracting the powers.

- In (b), subtracting the powers gives $2 - (-3)$. Recalling negative numbers from Module 1, this gives $2 + 3 = 5$. Remember that 10^5 is just shorthand for 1×10^5 hence the alternative answers. Note, however, that to write just 10^5 is more usual.

Q11 6×10^6 cells μl^{-1}.

- 'μ' means 'micro'. 'Micro' always means one millionth i.e. 10^{-6}. So a μl is a micro*litre* which is one millionth of a litre. It follows that there are one million (10^6) μl in one litre.

- In 5 litres blood there are 3×10^{13} red blood cells. In 1 litre there are $(3/5) \times 10^{13}$ cells i.e. 6×10^{12} cells. In 1 μl, $6 \times 10^{12} \div 10^6$ cells. This equals 6×10^6 cells μl^{-1}. Remember that the power '–1' means the same as 'per'.

Q12 (a) 10^{28} molecules m^{-3} (per m^3)
(b) 10^{19} molecules μl^{-1} (per μl).

- You know that 10^6 cm^3 = 1 m^3 (Workbook, Module 3, Q10 ANSWERS). Also, as there are 10^3 cm^3 in 1 litre, it follows that there are $10^6/10^3 = 10^3$ litres in 1 m^3. There are 10^{25} molecules in 1 litre, so there are $10^3 \times 10^{25}$ molecules in 1 m^3. Now divide 10^{25} by 10^6 to get the number of molecules in one μl.

Q13 0.037 4 g DNA (3.74×10^{-2} g DNA).

- Note that the unit of mass (grams, g) *must* be shown in your answer; 0.037 4 alone is wrong.

- Each human zygote contains 6.80×10^{-12} g DNA. The 5.5 billion people currently living grew from 5.50×10^9 zygotes. Thus the mass of DNA that coded for these was: $6.80 \times 10^{-12} \times 5.50 \times 10^9$ g. Note that all living people were coded for by less than a pin-head's mass (0.037 4 g) of DNA!

Q14 6.5 seconds.

- The sound travels a total distance of 2.2 km (= 2 200 metres). Its speed is 340 $m\,s^{-1}$. There are 2 200/340 'lots of 340 m' in the total distance. Thus the sound will take 2 200/340 seconds to cover it. (6.470 588 235 s = 6.5 s to 2 sf).

- The method above is the 'common sense' way of using the equation:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You may be able to see that this equation can be rewritten as:

$$\text{time (s)} = \frac{\text{distance (m)}}{\text{speed (m s}^{-1}\text{)}} = \frac{2\,200\text{ m}}{340\text{ m s}^{-1}} = 6.47\text{ s}.$$

This is what we had by the 'common sense' method anyway, so—if you have difficulty seeing how the equation can be rearranged in this way—just use the common sense way. (Rearranging equations is explained in Module 8).

Q15 (a) 4 (b) 8/3 or 2.67 (c) 1 000 or 10^3 (d) 1 000 or 10^3 (e) 10^{-9} (f) 5×10^{-4} (g) 2×10^5.

- The reciprocal of a number is 1 divided by that number. This means that to get the reciprocal of a simple fraction, you just 'turn it upside down'. This works with (a) to (c) in a straightforward way.

- As noted in the question, a simple rule for the reciprocal of powers of ten is to change the sign of the power. This works well for (d) and (e). It also works for (f) and (g) provided you also take the reciprocal of the whole number: so reciprocal of 2×10^3 is 0.5×10^{-3} which is 5×10^{-4} in scientific notation.

- For any problem the easiest method of taking reciprocals is to press the $1/x$ key on your calculator.

Q16 (a) 26 000 times. This is 2.6×10^4 in scientific notation. (b) 87 years.

- In (a), divide the distance to Andromeda by the distance to Sirius: $2.1 \times 10^{22} \div 82 \times 10^{16}$ (25 609 is 26 000 to 2 sf).

- In (b), you need to turn the speed of 3.0×10^8 $m\,s^{-1}$ into units of $m\,yr^{-1}$. Do this by multiplying by 'the number of seconds in a year'. Remembering that there are 60 seconds in a minute, 60 minutes in an hour etc, multiply the speed in metres per second by ($60 \times 60 \times 24 \times 365$) to give speed in metres per year. This gives 9.4608×10^{15} $m\,yr^{-1}$. Now treat this exactly as you did the speed of sound problem in Q14, i.e. time = distance/speed. So time to travel to Sirius (yr) = distance to Sirius (m)/speed of ship ($m\,yr^{-1}$). This gives 86.67 years—87 years to 2 sf.

MODULES 5/6 ANSWERS AND COMMENTS

Q1 (a) (i) silicon and oxygen (ii) none (iii) silicon and oxygen.
(b) (i) tin and lead (ii) tin and lead (iii) none
(c) (i) gold (ii) gold (iii) none.

Q2 (a) to (e) are correct descriptions of a nitrogen atom, (f) is incorrect.

- (a) and (b) are true for all atoms.

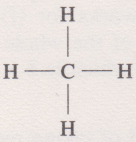
- Table 6 of Modules 5/6 gives the relative mass of nitrogen as 14. This means that one atom of nitrogen has 14 times the mass of one atom of hydrogen. (Note in passing that this simple definition of what is properly termed *relative atomic mass* has nowadays been replaced with a related but more sophisticated definition that you may meet in later courses e.g. the *Science Foundation Course*.)

- A nitrogen atom has uniquely seven protons and seven electrons. This is what makes it nitrogen: see Modules 5/6, Table 4.

- Nitrogen often forms three covalent bonds (Modules 5/6, Table 5). It *can* form other numbers, especially five: you may meet this in later courses e.g. in the *Science Foundation Course*.

- No atomic nucleus contains electrons.

Q3 Answers to parts (a), (b) and (c) are combined in the following Table.

H_2	H – H	hydrogen	gas	element
O_2	O = O	oxygen	gas	element
N_2	N ≡ N	nitrogen	gas	element
Cl_2	Cl – Cl	chlorine	gas	element
H_2O	H–O–H	water	liquid	compound
CH_4		methane	gas	compound

Q4 (a) Carbon atoms (2), hydrogen atoms (6) and an oxygen atom (1).

(b) C_2H_6O (sometimes written C_2H_5OH thereby showing that the last hydrogen is bonded to oxygen).

(c) C has 4 covalent bonds, hydrogen has 1 and oxygen has 2.

(d) C has a relative atomic mass of 12, H has a relative atomic mass of 1, oxygen has a relative atomic mass of 16 (see Modules 5/6, Table 6).

(e) Relative molecular mass of ethanol is 46.

- Modules 5/6 teach the concept of 'chemical formula'. In fact, there are different 'levels of detail' at which a chemical formula can be given. A simple statement of the numbers of each kind of atom per molecule is called (for covalent compounds) a *molecular formula* (e.g. H_2O). A simple diagram which shows how the atoms are connected by covalent bonds is called a *structural formula* (e.g. H—O—H). This is extra detail: the key thing is to know about 'chemical formulae'.

- Atoms, ions and molecules all have masses that can be expressed *relative* to that of a hydrogen atom (with H = 1). The proper term for the relative mass of an atom is (not surprisingly!) relative *atomic* mass. Similarly, chemists often speak of relative *molecular* masses of covalent molecules. It is fine, however, to stick with the more general term 'relative mass'.

- To get the relative mass of a molecule, add up the relative atomic masses of the atoms in it. (So, for ethanol, $46 = 2 \times 12 + 6 + 16$).

Q5 The ratio is 1 : 2 (or 0.5 : 1 or just 0.5). Note that 14 : 28 is correct but not really acceptable as ratios should be reduced to their lowest terms).

- From Table 6, relative masses (more correctly, relative atomic masses) of N and Si are 14 and 28, hence the answer.

Q6 (a) 44 (b) for the structural formula of propane see Figure 17.

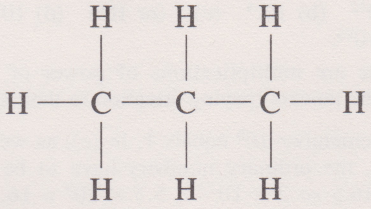


FIGURE 17 The structural formula of propane.

- Relative mass (or relative molecular mass) of propane is $3 \times 12 + 8 \times 1 = 44$.

- Give carbon 4 'hooks' and hydrogen 1 'hook'—then join them together (4 covalent bonds and one covalent bond, in chemical language), the *only* way to write the structure is that shown above!

Q7 (a) carbon disulphide, CS_2
(b) hydrogen sulphide, H_2S (c) 1
(d) carbon tetrabromide, CBr_4.

- S_2C is simply not conventional. If you wrote that, you are not wrong but, next time, recall the conventional way. Many subjects have aspects that depend simply on familiarity and memory.

- Logic might suggest the name dihydrogen sulphide. (Alternatively, we could have written SH_2 and called it sulphur dihydride!) *In fact*, chemists call it 'hydrogen sulphide' and write it: H_2S.

Q8 The atoms in water are joined by covalent bonds and so form separate molecules of H_2O. In contrast, the ions of Ca^{2+} and O^{2-} are attracted to each other (and to neighbouring Ca^{2+} and or O^{2-} ions) to form a three–dimensional arrangement of ionic bonds.

Q9 (a) sodium chloride (b) calcium oxide (c) potassium iodide (d) calcium bromide (e) zinc oxide (f) magnesium oxide (g) aluminium oxide.

- If you suggested calcium dibromide in (d), you are wrong only from the standpoint of convention.

Q10 (a) Al^{3+} (b) I^- (c) Br^- (d) Zn^{2+} (e) Mg^{2+}.

Q11 The answers to this question are shown in the completed Table at the end of this Section.

- Note that the ratio of the numbers of + and – ions is such that the total number of – charges exactly equals the total number of + charges. An ionic substance has a *net* charge of zero.

- The formula relates exactly to the ratio.

- Where two (or more) lots of complicated ions like nitrate or sulphate are needed to achieve zero net charge, *brackets* are needed to show two (or more) lots of the *entire* ion. The last three entries in the far right column illustrate this.

- If you were able to complete the bottom row, you really are fluent in the skill of writing ionic formulae.

Q12 Ionic compounds consist of positive ions and negative ions. Each of these is attracted to *all* neighbouring ions of opposite charge. For this reason there is no precise grouping of atoms into molecules as occurs in covalent compounds.

Q13 $H_2 + I_2 = 2HI$
$2Co + O_2 = 2CoO$
$2As + 3Cl_2 = 2AsCl_3$.

Q14 $C_3H_8 + 5O_2 = 3CO_2 + 4H_2O$
$MgO + 2HCl = MgCl_2 + H_2O$
$CH_4 + 4Br_2 = CBr_4 + 4HBr$.

Q15 0.001 g cm^{-3} or 1 mg cm^{-3}.

• If there is 1 g in 1 litre, there is 1/1 000 g in 1 cm^3. (Hence 0.001 g cm^{-3}. As 1 mg is 1/1 000 g, the answer is also 1 mg cm^{-3}).

Q16 6.3 mg l^{-1}

• 1.6 mg in 256 cm^3 means that there are $1.6/256 = 6.25 \times 10^{-3}$ mg cm^{-3} (6.25 mg per cm^3).

In 1 litre there are 1 000 cm^3 hence the concentration is $6.25 \times 10^{-3} \times 10^3 = 6.25$ mg l^{-1}. *But* the mass of sulphate ion is given to 2 sf and the volume of mineral water is given to 3 sf. The answer should be expressed to the *smaller* of these i.e. to 2 sf. (6.25 is 6.3 to 2 sf).

Q17 (a) CO_3^{2-} (b) 1 : 1 (c) 60

• Relative mass of CO_3^{2-} is $12 + 3 \times 16 = 60$.

Q18 (a) 31 g (b) 6.2 mg l^{-1} phosphorus.

• There are 31 'relative mass units' of (or 'parts' of) phosphorus in every 95 'relative mass units' of phosphate ion. So in 95 *grams* of phosphate there must be 31 *grams* of phosphorus.

• From (a), in 95 parts by mass of phosphate ion there are 31 parts by mass of phosphorus. So, in 1 part by mass of phosphate ion there are 31/95 parts by mass of phosphorus. So, in 19 mg of phosphate ion there are $19 \times 31/95 = 6.2$ mg phosphorus.

So, expressed as phosphorus, 19 mg l^{-1} phosphate ion is 6.2 mg phosphorus l^{-1}.

Completed Table for Q11.

Name of ionic compound	Identity of positive ion	Identity of negative ion	Ratio of no. of positive ions to no. of negative ions.	Formula of ionic compound
sodium chloride	Na^+	Cl^-	1 : 1	$NaCl$
calcium chloride	Ca^{2+}	Cl^-	1 : 2	$CaCl_2$
sodium nitrate	Na^+	NO_3^-	1 : 1	$NaNO_3$
sodium sulphate	Na^+	SO_4^{2-}	2 : 1	Na_2SO_4
calcium nitrate	Ca^{2+}	NO_3^-	1 : 2	$Ca(NO_3)_2$
aluminium nitrate	Al^{3+}	NO_3^-	1 : 3	$Al(NO_3)_3$
aluminium sulphate	Al^{3+}	SO_4^{2-}	2 : 3	$Al_2(SO_4)_3$

MODULE 7 ANSWERS AND COMMENTS

Q1 (a) dependent, *y*, independent, horizontally, *x*
(b) scale, variables (c) units, quantity, title (d) axes, origin.

• No other arrangement that is both correct and grammatical, is possible. ('Variables' and 'quantity' could be interchanged except that, in these sentences, one has to be singular and the other has to be plural!)

Q2 The completed Table is shown below.

Expt.	Variable	D or I?	Units?	Graph no?
Expt A	solubility	D	g l^{-1}	(ii)
	temperature	I	°C	
Expt B	time	I	min	(i)
	temperature	D	°C	
Expt C	train speed	I	km hr^{-1}	(v)
	stopping time	D	min	

• The independent variable is the one you choose as experimenter. The dependent variable follows as a consequence—it is the one you measure. In A, you choose various temperatures and see how much glucose dissolves. In B, you choose times at which to measure the water temperature. In C, you choose a train speed and then, on applying the brakes, you time how long it takes to stop completely.

• The only way that the units listed can be matched with the quantities measured is as shown. Note both 'minutes' and '°C' are used twice—in each case, once as the independent variable and once as the dependent variable.

• As regards the sketch graphs, consider the following. For A, the solubility experiment, the only line that increases then flattens ('plateaus' is the technical term) is (ii). For B, the only line that is level throughout is (i) and this therefore reflects the constancy of the temperature of the insulated tank. For C, you are told that stopping time increases as the speed of the train increases (at the moment of application of the brakes). This could suggest graph (v) or possibly (ii). General knowledge about any wheeled vehicle (cars, trains etc.) should tell you that the faster it goes the longer it takes to stop when you brake. Hence, (v) is the reasonable choice.

Q3 (a) average mass of voles on the *y* axis (vertical); age of voles on the *x* axis (horizontal). (b) We chose 1 cm = 2 g and 1 cm = 10 days with the paper oriented with days horizontal across the 8 cm width of the paper. (c) mass/g and time/days (*y* and *x* respectively). (d) Plot of average mass of voles against age (or something similar).

• The oblique stroke followed by a unit (g or days in this example) is the conventional way of labelling an axis with variable and its units. You also saw it in the Table for this question. You should follow this convention.

Q4 (a) See our graph in Figure 18 (b) Growth is relatively slow to start with, then is more rapid. It finally becomes slower, reaching a plateau at about 50 days.

- As growth is probably continuous and has different (but unknown) values in between the actual dates chosen for measurement, the best smooth line representing this continuous growth should be estimated and then drawn so that it passes through (or near to) these points.

- Not all points have to be on the drawn line. For example, one or more could be wrong because of an error in measurement.

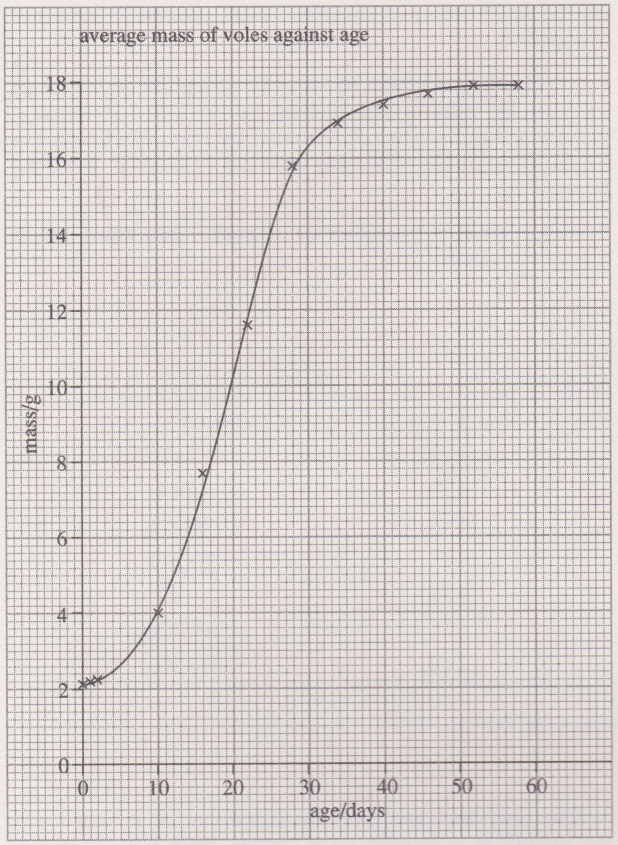


FIGURE 18 Graph of average mass of voles against age.

Q5 (d).

- Things grow exponentially if they increase by a constant 'multiplication factor' in equal intervals of time, e.g. if the number of organisms in a jar of pond water increase every day from 2 to 4, from 4 to 8, from 8 to 16 etc, the number *increases by a factor of 2 every day* i.e. the number doubles every day.

- The word 'factor' *means* 'number you multiply by' so, correctly, the word 'multiplication' in the previous paragraph is unnecessary.

- Test whether a series of measurements taken at constant time intervals represents exponential growth by dividing the first into the second, the second into the third, the third into the fourth, and so on. If the number (i.e. factor) yielded by each division is always the same, then the 'multiplying factor' is always the same, and growth is exponential. In the example above, the factor is 2. ($4 \div 2 = 2$, $8 \div 4 = 2$, $16 \div 8 = 2$, etc.).

- It is only in (d) that there is a constant factor. This time, the factor is 3. (10.5/3.5 = 3, 31.5/10.5 = 3 etc). In (a), there is a constant *difference* between numbers (of 2)—there is no constant factor to denote exponential growth!

Q6 (b) (c), (d), (see note below about (c)).

- In (a), although there is a contant *difference* (of 2.9) between one number and the next, there is no constant *factor*; thus there is no exponential change. In (b) the constant factor is 4. In (c) the constant factor is 0.5. Although a series with a constant factor of 0.5 could be called 'exponential change', it is more often termed '*exponential decrease* or decay'. (d) is exponential; the fact that the numbers are negative does not affect whether or not a series of numbers is expontial. Imagine an overdraft at the bank. The constant factor in (d) is 3.

Q7 The completed Table is shown below.

Year	1	2	3	4
No. of cases	5	15	45	135

The question tells you that the factor is 3. Without using a calculator, it is easy to see that $3 \times 5 = 15$, $3 \times 15 = 45$, and $3 \times 45 = 135$.

Q8 If your calculator is able to 'multiply by a constant' —and most can —it is very useful to know how to do this. Experiment with your calculator until you can do it.

- Note, in passing, that calculators that can 'multiply by a constant' can also 'divide by a constant', 'add a constant amount' and 'subtract a constant amount'. Once you have sorted out how to multiply by a constant, experiment with easy numbers on your calculator until you can divide, add and subtract a constant, too. All calculators that multiply by a constant by two presses on the × key do the other operations in a similar way: press the correct operation key (÷, +, or −) TWICE IN SUCCESSION.

Q9 As the number of times you want to 'multiply by a constant' increases, the more and more tedious it becomes. The x^y key lets you do this very quickly. The number of cases in year 4 is calculated by $5 \times 3 \times 3 \times 3$. This is, of course $5 \times (3 \times 3 \times 3)$ which is 5×3^3. Do this by entering 5, ×, 3, x^y, 3 and =, and 135 is displayed.

Q10 23 914 845 cases of the disease after 15 years.

- Enter 5, × , 3, x^y, 14 and = and the answer above is shown. (Note: 14 not 15 because you start at Year 1.)

Q11 If you plotted these six points correctly, you should get a large tick on your graph paper when you join each point to the next. If you do not get a tick, you have done it wrongly and you should check your points. It is important to remember that when pairs of values are given for points the x value is given first, followed by the y value.

MODULE 8 ANSWERS AND COMMENTS

Q1 $M_{av} = \dfrac{3.0 + x + y + z}{6}$ kg

- You'll come across all kinds of subscripts in algebra m_1, t_o, M_{av} etc. Averages are calculated by adding together all values and dividing the total by the number of values. The fact that some are numbers and some are letters makes no difference.

Q2 (a) ghd cm^3 (b) 5h cm (c) 10g cm (d) 50gh cm^2

- Volume = length × height = depth = g × h × d cm^3

Q3 $\rho = \dfrac{M \times 10^6}{V}$ kg m^{-3}

- Density is mass/volume: grams per cm^3 or kg per $metre^3$ (g cm^{-3} or kg m^{-3}). Units in the two alternatives are not normally mixed e.g. *not* g per $metre^3$. So, convert V cm^3 into $V \times 10^{-6}$ m^3.

Hence $\rho = \dfrac{M \text{ kg}}{V \times 10^{-6} \text{ m}^3}$

As '1 over 10^{-6}' is 10^6, and as '1 over m^3' is m^{-3}, we get:

density (ρ) is $\dfrac{M \times 10^6}{V}$ kg m^{-3}.

(Note: instead of writing '1 over 10^{-6} is 10^6' we could equally well have written 'the inverse of 10^{-6} is 10^6' or 'the reciprocal of 10^{-6} is 10^6'. Use whichever form of words suits you best.)

Q4 d = (vt/2 000) km.

- The distance travelled is 2d km. The speed given is v m s^{-1}. Thus speed is v × 10^{-3} km s^{-1}. You know that speed = dist/time. Therefore, speed × time = distance. Thus, v × 10^{-3} km s^{-1} × t sec = 2d km.

So, divide both sides by 2:

$d = \dfrac{vt \times 10^{-3}}{2}$ km

This is an acceptable answer as it stands. It is equally neat to take the 10^{-3} from the numerator to the denominator and write vt/2 000 km as above.

Q5 (a) 10^{x+y} (b) 10^{m-n} (c) 10^{w+b-c}
(d) 10^{-q} (e) 10^{t} (f) 10^{0} (= 1) (g) $6 \times 10^{p+q}$
(h) $1.2 \times 10^{3w+1}$

- Apply the rules in Module 4, Sections 2.3 and 2.4.

- (d) and (e) amount to 'take the reciprocal of …' (that is what '1 over something' is).

- (f) is 1. The way '1' is written as a power of 10 is 10^0 (see Module 4, Section 2.6).

- In (g) the ordinary numbers must be multiplied together (2 × 3) then the rest is multiplied using the power rules.

- In (h) the addition of the powers gives w + w + w which is 3w. The multiplication of 1 × 3 × 4 gives 12. If you got 12 × 10^{3w} you are nearly there. To put this in scientific notation, we need 1.2 × 10 not 12. So, we need to increase the power by another 1 thus giving a power of (3w + 1). If you find this puzzling, try it with real numbers—for example putting w = 2.

Q6 $\dfrac{B \times 10^{23}}{3}$ molecules.

- 18 g water contain 6 × 10^{23} molecules. 1 g contains (6/18) × 10^{23} molecules = 1/3 × 10^{23} molecules ∴ B g water contain B times as much as this. Hence the answer.

Q7 (w + x − 100) g oxygen.

- Notice the optional use of brackets to make clear the g of grams is applying to the whole of (w + x − 100) g

- The word equation is:

hydrocarbons + oxygen = carbon dioxide + water
100 g ? x g w g

As nothing else is involved and the masses on each side must equal each other, (100 + ?) grams must equal (x + w) grams. From this, ? must be (x + w − 100) g.

- When doing problems in algebra, it is common to choose for yourself another letter to represent the 'unknown thing' you are trying to find. This problem is a good example. You could have said 'let the mass of oxygen be y'. Then you would have avoided the clumsy '?' because the word equation would have been:

hydrocarbons + oxygen = carbon dioxide + water
100 g y g x g w g

You now write the equation 100 + y = x + w and rearrange to get y as the subject of the equation.

Q8 (a) $x = y - 9$ (b) h = g − 2x (c) $j = \dfrac{m-k}{2}$

(d) $c = (a-b)^2$ (e) $u = \sqrt{m-n}$ (f) r = 7q

(g) e = 8/d (h) $w = \dfrac{1}{9-x}$ (i) g = 5(f + k)

(j) $v = \dfrac{3}{y} - 2$ (k) $x = \dfrac{y-c}{m}$ (l) $v = \dfrac{s}{mw}$

- All these rearrangements are done by carefully applying the rule of 'doing to one side exactly what you do to the other'.

- If you find it hard to believe any of these answers, choose simple real numbers for the letter(s) on the right hand side and work out the numerical value of the subject. Then put this into the rearranged equation and if it works the rearrangement is right! For example, in (a), let's choose $x = 1$. Clearly $y = 10$. Now look at what you are told is the answer: $x = y - 9$. Put in the calculated value of 10 for y and you get $x = 10 - 9 = 1$. But that's what we chose, so the answer checks. If you are confident of your method and answer, DO NOT waste time by checking your work in this very time consuming way.

- Sometimes, 'getting rid of the fraction' is not the best first step. In (i) it is best to add k to both sides (giving f + k = g/5) and *then* get rid of the fraction by multiplying both sides by 5. Here, you have to put in brackets to show that all of (f + k) is to be multiplied by 5—giving the answer of g = 5(f + k).

- In (k) the equation is $y = mx + c$. We are to make x the subject of the equation. Take c from each side so $y - c = mx$. Now divide both sides by m to get $(y - c)/m = mx/m$. The letters m can be cancelled out on the right giving us $(y - c)/m = x$ or $x = (y - c)/m$.

Note that $x = (y - c)/m$ is exactly the same as $x = \dfrac{y-c}{m}$ The brackets are added to show that the *whole* of $y - c$ is divided by m. In writing (as distinct from print where saving space is important) it is better to use the horizontal line method.

- Note finally that some of these are quite *difficult* and are included for those who enjoy this kind of thing! So long as you have the idea and can do some of the obviously more straightforward ones, that is fine.

Q9 (a) m = 4n (b) $x = y - 108$ (c) $q = \dfrac{12-r}{7}$

(d) t = s − 7u (e) $h = \dfrac{(g-6)^2}{3}$ (f) $h = \dfrac{g-6}{\sqrt{3}}$

(g) $c = \sqrt{\dfrac{a-b}{2}}$ (h) $m = \dfrac{xs-4}{n}$ (i) $b = \dfrac{1}{a^2}$ or a^{-2}

- As before, some of these are difficult. Let's compare (e) and (f). To do (e), subtract 6 from each side and get:

$g - 6 = \sqrt{3h}$.

Now square *all* of each side and we get $(g - 6)^2 = 3h$. Now divide each side by 3 to get the answer above. (Remember, if you 'square the square root of 4', you just get 4. So, squaring the square root of 3h gives 3h.)

Now try (f) which apparently differs only slightly. As before, subtract 6 from each side to get $g - 6 = h\sqrt{3}$. Now divide each side by $\sqrt{3}$ to get the answer above.

- In (i) remember that $1/a^2 = a^{-2}$.

Q10 (a) x = 4 or –4 (b) $y = 49$ (c) q = 12
(d) m = 15 (e) n = 4 or –4 (f) r = 64 (g) r = 80

• As ever, you must do the same thing to both sides of the equation. So, in (a) take the square root of both sides but remember $x^2 = (-4)^2$ or $x^2 = (4)^2$.

• In more complicated ones, you need to rearrange the equation step by step to get the letter whose value you are trying to find *as the subject* of the equation.

Q11 M kJ = (H + S) kJ

• This is a scientific statement about energy conversion (and the Law of Conservation of Energy) and not just algebra. The units (kilojoules) must be included in your answer as shown. Note, the brackets just make it easy to see that 'kJ' applies to (H + S).

Q12 (a) 1.2ab J (b) 120 J

• The units must be shown.

• As the question tells you, to get kinetic energy (KE) in joules using the equation

$KE = \frac{1}{2} M v^2$, you must have M in kg and v in m s^{-1}.

Get M in g by applying the equation:

Density (g cm^{-3}) = mass (g)/volume (cm^3).

So 2.4 g cm^{-3} × 10ab cm^3 = 24ab grams. This is: 24ab/1 000 kg.

Now apply $KE = \frac{1}{2} M v^2$ and get $KE = \frac{1}{2} \times \frac{24ab \times 100}{1\,000}$ giving the answer of 1.2ab J.

• Substituting a = 25 and b = 4.0 into this gives 120 J.

Q13 $d = \sqrt{\frac{kI}{v}}$ cm

Q14 (a) Q(A + B + C) g (b) $\frac{A}{(A + B + C)} \times 100\%$

This can also be written $\frac{100A}{(A + B + C)}$ %

• Total mass in *one* litre is (A + B + C) g. So there will be Q times as much in Q litres.

• Consider just 1 litre. In this there are A g of nitrate ion. This is the fraction A/(A + B + C) of the total mass (A + B + C) g. Multiply the fraction by 100 to get the percentage. The same percentage applies to the residue from Q litres (which is what the question asks about).

Q15 (a) It does balance: 3 C atoms on each side, 8 H atoms on each side and 10 O atoms on each side. (b) $C_xH_y + (x + y/4) O_2 = xCO_2 + y/2 H_2O$

• There must be xCO_2 as there are x carbon atoms. That bit is easy! There must be y/2 lots of H_2O because this gives y atoms of hydrogen on each side. To make the oxygens balance, we need x lots of O_2 and then another y/2 lots of O. But O only comes as O_2 so y/2 lots of O is really y/4 lots of O_2. Altogether, we have x + y/4 lots of O_2.

Q16 (a) yes (b) $10Q^{10}$ rabbits

• The definition of 'exponential growth' *is* that something increases by a constant factor in equal periods of time. So this example is undoubtedly exponential growth. ('Factor' is a number you multiply by.)

• Jan 1980 to Jan 1990 is 10 years. In Jan 1981, there would be 10Q rabbits. In Jan 1982 there would be Q × 10Q rabbits = $10Q^2$. In Jan 1983: Q × Q × 10Q = $10Q^3$ rabbits. Hence after 10 years there are $10Q^{10}$ rabbits.

Q17 (a) 1/ab (b) b/a (c) a/b (d) C^y (e) s (f) m^2

• Recall the meaning of reciprocal from Module 4. If you have any difficulty doing these, put simple numbers in place of the letters. For example, how can we show that the reciprocal of 1/ab is ab? Remember that ab is shorthand for a × b. So, think about the reciprocal of 1/(2 × 5). Clearly, this is the reciprocal of 1/10. And this is 10—which *is* 2 × 5. So, the reciprocal of 1/ab must be ab.

MODULE 9 ANSWERS AND COMMENTS

If any of your answers is given to more significant figures than shown below, it is likely that you are claiming more accuracy than the data in the question warrants.

Q1 (a) 29 cm (b) 327 m (c) 9.401×10^{11} m
or 9.400×10^{11} m if you use the π key.

• In each case, apply $C = 2\pi r = \pi d$. Remember $2r = d$.

• If you wish, you can use the π button on your calculator to save entering the digits of 3.142 separately. If you are uncertain of how to use that button, you could try part (a) of Q1 without the π button, and then with it. Using the button means that you are taking π as 3.141 592 654. Do not make the mistake of giving your answer to this number of figures, however!

• If you specifically *want* to use π to just 4 figures in a series of calculations, you could save a bit of time by entering 3.142 then pressing M IN. This places 3.142 into the calculator's memory. Every time you do a problem that requires you to multiply by this value of π, you press × then MR where MR is 'memory recall'. For example, to solve problem (a), you enter 2 × 4.6 × MR =.

Note that whenever you want to get rid of unwanted material in your calculator's memory, just press 0 then M IN.

• In (c), 9.401×10^{11} m is the distance from the Sun to the Earth. Assuming the Earth's orbit is circular (but it isn't quite), the circumference that you calculated is the distance the Earth travels in one year.

Q2 (a) 40 118 km (b) 40 527 km (c) 1.010 2 (or 1.010 2 : 1)

• Diameter of circle (of the plane's journey) = 12 756 + 7 + 7 = 12 770. Diameter of circle (of the satellite's journey) = 12 756 + 72 + 72 = 12 900. These diameters when multiplied by 3.141 6 give the circumferences to 5 sf.

Note that we are told the heights of the plane and satellite are *exactly* 7 km and 72 km above sea level. This means that the number of significant figures to which the answer should be expressed is not limited by the accuracy of these heights. In fact, both π and the Earth's diameter are given to 5 sf, so the answer is given to 5 sf, too.

• In (c), recall from the Module 2 ANSWERS (Q12) that ratios can be expressed in several ways—as the answer above shows.

Q3 Ratio = 1 (1 : 1).

• The Equator is the circumference of Earth bisecting the northern and southern hemispheres. The Greenwich meridian is (like all lines of longitude) the circumference of the Earth through the N and S poles. The ratio of two equal things is necessarily 1. (Because the Earth bulges at the Equator, this length is slightly longer.)

Q4 (a) 3.141 6 (b) This is the value of π to 5 sf. You would expect this as 'circumference ÷ diameter of a circle' always equals π. ($C = \pi d$. So, rearranging this equation, $C/d = \pi$).

Q5 (b) and (d) only.

• For (a), (b) and (c), see Module 9. In (d), because lines of longitude all have the same circumferences, it follows ($C = \pi d$) that they all have the same diameter. In (e), because different lines of latitude have different lengths, it follows that they have different diameters.

Q6 (a) 10/11 (b) 2 (c) 14 cm.

• The safest way to deal with 'triple decker fraction' is to recognize that we are dividing by the 'bottom fraction', see Module 9, Section 1.3.

So, $\dfrac{40}{14 \times \frac{22}{7}}$ means $\dfrac{40}{14} \div \dfrac{22}{7}$

When you use the rule which says 'change the ÷ into × and turn the fraction that you are dividing by upside–down', this becomes:

$\dfrac{40}{14} \times \dfrac{7}{22}$ After cancelling, this then gives $\dfrac{10}{11}$

• When you are really confident, you can simply 'whisk' the bottom of the triple decker up to the top—the '7' in this example.

• Note that a 'triple decker fraction' cannot be handled in the way described above, if the multiplication sign is replaced by a plus or minus sign.

• In (c), remember that $C = \pi d$, so that, rearranging, $C/\pi = d$.

Thus, $d = \dfrac{44\text{ cm}}{\frac{22}{7}} = \dfrac{44\text{ cm} \times 7}{22} = 14\text{ cm}.$

Q7 (a) 66 cm^2 (b) 16 300 or 1.63×10^4 m^2
(c) $\pi x^2/4$ cm^2.

• Apply the equation: area = πr^2. In (b), remember that 102 m has to be divided by 2 before you start. In (c), the same applies to x cm. So area = $\pi(x/2)^2$ cm^2 = $\pi x^2/4$ cm^2

• As before, you can enter 3.142 into the calculator's memory to save effort. If you do this, you then enter the radius, square it, and finally multiply by 'memory recall'.

Q8 (a) The answers are shown in the Table below.
(b) $r_1^2 : r_2^2 : r_3^2$ (c) 408 : 22 : 1

	(1) Diameter/ m	(2) Diameter/ mm	(3) Area/ mm^2
Artery	5.0×10^{-4}	5.0×10^{-1}	2.0×10^{-1}
Arteriole	1.2×10^{-4}	1.2×10^{-1}	1.1×10^{-2}
Capillary	2.5×10^{-5}	2.5×10^{-2}	4.9×10^{-4}

• Diameters in column (2) are those in column (1) × 10^3.

• To get the areas in column 3, halve the diameters in column 2 to obtain the radii (plural of radius) and then apply A = πr^2. Answers are to 2 sf.

• Scientific notation has been used throughout the Table. Answers in ordinary notation, 0.50, 0.12, 0.25 etc., are fine so long as you show 2 sf; thus, you must have 0.50 and not 0.5

• Units must not be written by the values as they are shown at the head of each column.

• The following is the answer to part (b). The ratio of the areas is $\pi r_1^2 : \pi r_2^2 : \pi r_3^2$, where r_1, r_2, and r_3 are the radii of the three blood vessels. The 'π' can be cancelled from all of these, so the ratio of the areas is the ratio of the values of each r^2 i.e. $r_1^2 : r_2^2 : r_3^2$.

• For (c), the meaning of the ratio is that an artery has 408 × the area of a capillary, and that an arteriole has 22 × area of a capillary. To get these ratios, you divide the smallest area into each of the others.

Q9 (a) 270 cm^2 (or, better, 2.7×10^2 cm^2)
(b) 32 700 m^2 (or, 3.27×10^4 m^2) (c) $12.57x^2$ cm^2.

• In (a) to (c), apply: surface area = $4\pi r^2$.

• It is better to use scientific notation as you can indicate the necessary number of sf—2 in (a), 3 in (b) and 4 in (c) —in an unambiguous way. In (c), the answer is given to 4 sf i.e. the same as π = 3.142. This is because 'x' cannot be assumed to be of limiting accuracy (it might be to 10 places for all we know!) so we use the four figures of π as our guide.

Q10 $100p^2 : q^2$

• The ratio is $4\pi(10p)^2 : 4\pi q^2$, where 10p and q are the respective radii—both now in millimetres. The '4π' on either side of the : sign cancel out. Then, squaring 10p to give $100p^2$ gives the answer shown.

• As any ratio can be expressed as 'something : 1', acceptable alternatives to the answer shown are $100p^2/q^2 : 1$ or simply $100p^2/q^2$. Ratios are frequently given in these different styles.

Q11 6.41×10^7 J m^{-2}

• Applying the formula, surface area = $4\pi r^2$ to the data gives $6.088\,140\,288 \times 10^{18}$. (If you have used the π button you get $6.087\,350\,988 \times 10^{18}$). Do not correct to three figures at this stage; just enter it in the memory by pressing M IN.

• To get energy per unit area, you need to enter the energy, and then divide by the area. So enter 3.9×10^{26}, press ÷, press MR then press =. Correct the displayed answer to 3 sf and express in scientific notation as shown above.

• Note that the answer to 3 sf is unaffected by your choice of π = 3.142 or of the value given by the π button.

Q12 (a) 4.1×10^2 cm^3 (b) 5.56×10^5 m^3
(c) 9.69×10 m^3 (or more usually 96.9 m^3).

• In each, apply volume = $4\pi r^3/3$. See Box 5 of Module 9 for how to do cubes using x^y key.

• In (c), first find radius of the sphere from the surface area of the sphere by rearranging area (of 102 m^2) = $4\pi r^2$. This equals 2.848 833 055 (keep this number on the calculator). Now apply the usual $4\pi r^3/3$.

Q13 1.30×10^6 times as great.

• The ratio is $4\pi R^3/3 : 4\pi r^3/3$ where R and r are the radii of Sun and Earth respectively. $4\pi/3$ cancel from each leaving a ratio of $R^3 : r^3$. But we are told that R is 109 times as great as r; mathematically, that is the same as saying $R = 109r$. So, we can *substitute* $109r$ in place of R, giving a ratio of $(109r)^3 : r^3$. This is $109^3 \times r^3 : r^3$. The r^3 now cancel, giving $109^3 : 1$. Using the x^y button to cube 109, and correcting to 3 sf, we get the answer above.

Q14 (a) 2.1×10^{-31} m^3 (b) 6.6×10^{57}.

- 10^{57} is the largest number in *Into Science*! It is vast!

- Calculate (a) by applying $4\pi r^3/3$. Notice how very small the volume is! Enter this answer in the memory.

- Now enter the volume of the Sun given in the question (1.4×10^{27} m^3), press ÷ then press MR (the volume of the hydrogen atom) and finally, press =. The display gives you the ratio of the volume of Sun : volume of H atom to a large number of sf! Correct this to the number of sf appropriate to your data—namely 2.

- It is normal to keep all intermediate steps in a calculation at the *greatest number of figures* that the calculator will allow. Only correct to the required number *right at the end*. If you correct too early, it will sometimes—incorrectly—alter the final figure of your final answer. This is explained in Module 3.

Q15 a = 35°, b = c = h = 105°, d = e = 75°, f = 145°, g = 40°, i = 55°.

- The figure tells us that the top and bottom lines are parallel to each other. Therefore, a = 35°.

- In the top triangle, 40° + 35° + b must equal 180°—because the sum of angles in *any* triangle is 180°. Hence b = 105°.

- But b = c as they are vertically opposite angles:so c = 105°.

- The two lines that slope upwards from left to right are also parallel to each other as both make an angle of 35° with the bottom line. Because they are parallel, c = h; so h = 105°, too.

- d = e as they are vertically opposite. But 360° (the circle centred on the point where the lines cross) is formed from b + c + d + e. But b + c = 210°, so:
d + e = 360° − 210° = 150°. So d = e = 75°. Alternatively, b + e = 180°, and since b = 105° then e = 75°.

- g can be found from 'sum of angles in a triangle', because we now know the other two angles.

- f is easily found as f + 35° must = 180° (angles in half a circle or a straight line).

- Finally, i is found by 'sum of angles in a triangle': the other angles are given as 35° and 90°.

Q16 (a) the Equator (b) eastwards (c) 26% (to 2 sf).

- In a map reference latitude is given first. This is 0° for both places, therefore the line of latitude is the equator.

- Now put a dot on Figure 28 at 174° E. This is a little to the left of the 180° longitude. Put a dot at 91° W. This is immediately to the left of the 90° W longitude. The number of degrees longitude between the dots is 6 + 30 + 30 + 29 (look at Module 9, Figure 28) = 95°.

- The whole span of the Earth is 360°, so the fraction is 95/360 and the percentage is 95/360 × 100.

MODULE 10 ANSWERS AND COMMENTS

Q1 (a) 67.4°, 90° and 22.6°
(b) AB^2 should equal $CB^2 + CA^2$ because Pythagoras' theorem applies to right-angled triangles:
27.04 = 23.04 + 4.00.
(c) 67.4° + 90° + 22.6° = 180°; this is expected as the three angles of any triangle add up to 180°.

Q2 (a) Since the first two angles of each of triangles ABC and XYZ are equal, and since the sum of all three angles in any triangle is 180°, it follows that the third angle in each triangle *must* be equal. (b) 'similar' (c) 4 : 3. (d) Because the triangles are similar, AC/XZ and CB/ZY must also be 4 : 3. ZY/CB is the reciprocal ratio—namely 3 : 4 (e) YZ = 1.5 cm; ZX = 3.6 cm (f) By Pythagoras' theorem: $XY^2 = YZ^2 + ZX^2$; so 15.21 = 12.96 + 2.25.

- In (c) you are asked about a ratio. The following revises ratios for the last time.

The ratio AB/XY is 5.2/3.9 (alternatively expressed 5.2 : 3.9). Top and bottom of the fraction can be divided by 1.3 to give 4/3 (alternatively 4 : 3).

There are yet other ways of expressing the same ratio: 4/3 : 1 or 1.33 : 1 or just 1.33! You can choose whichever way you prefer to express a ratio, unless of course the question gives you no choice (as was the situation in part (c)).

It is important to follow the rule that the number on the *left* of the ratio sign goes on *top* of the fraction. In the example: the ratio of AB to XY, AB goes on *top* of the fraction.

- In (d), the ratio ZY/CB is the reciprocal of the ratio CB/ZY. In small whole numbers this ratio is 3 : 4. The other forms are: 3/4 or 3/4 : 1 or 0.75 or 0.75 :1.

Q3 (a) $\angle i = \angle R$ (b) Isosceles: because two of its angles are equal to each other (c) TM = SM because the sides opposite the equal angles are equal.

- In (a), ∠MTN + 90° = ∠MSN + 90°. So, the third angle in each of triangles TMN and SMN must also be equal; thus $\angle i = \angle R$.

Q4 The new triangle is an equilateral triangle—all sides are equal and all angles equal to 60°. This follws from the fact that ∠TMS = $\angle i + \angle R$ = 30° + 30° = 60°. As ∠TMS is 60° and the other two angles are equal to each other, it follows that each of the other two angles is also 60°.

Q5 (a) XY (b) For ∠X, ZY is opposite and XZ is adjacent (c) For ∠Y, XZ is opposite and ZY is adjacent.

Q6 The answers are shown in the completed Table:

Name of trig ratio	Ratio of sides	Ratio to 3 sf	∠X to 3 sf
sin X	$\frac{\text{opp}}{\text{hyp}} = \frac{ZY}{XY} = \frac{1.5}{3.9}$	0.385	22.6°
cos X	$\frac{\text{adj}}{\text{hyp}} = \frac{XZ}{XY} = \frac{3.6}{3.9}$	0.923	22.6°
tan X	$\frac{\text{opp}}{\text{adj}} = \frac{ZY}{XZ} = \frac{1.5}{3.6}$	0.417	22.6°

Q7 The answers to part (a) are shown in the completed Table:

Name of trig ratio	Ratio of sides	Ratio to 3 sf	∠Y to 3 sf
sin Y	$\frac{\text{opp}}{\text{hyp}} = \frac{XZ}{XY} = \frac{3.6}{3.9}$	0.923	67.4°
cos Y	$\frac{\text{adj}}{\text{hyp}} = \frac{ZY}{XY} = \frac{1.5}{3.9}$	0.385	67.4°
tan Y	$\frac{\text{opp}}{\text{adj}} = \frac{XZ}{ZY} = \frac{3.6}{1.5}$	2.40	67.4°

(b) If you compare the two Tables, sin X = cos Y. Similarly cos X = sin Y.

Also, ∠X + ∠Y = 90°.

The explanation is fairly simple: the side opposite to ∠X is *also* the side that is adjacent to ∠Y; the side adjacent to ∠X is *also* the side that is opposite to ∠Y. This can be seen from Figure 12.

(c) For interest, you could also note that the final relationship that can be seen from the Tables is that tan X = 1/tan Y and tan Y = 1/tan X. Confirm this using the reciprocal key on your calculator (enter 0.417, press 1/*x* and 2.398 should appear, or 2.40 to 3 sf).

All these relationships *are always true* in any right–angled triangle.

Q8 (a) length is 45.5 cm (b) 0.202 (c) 44.6 cm
(d) see below.

- Your sketch should be of triangle with a horizontal line for the floor distance, the hypotenuse as the sloping distance, and the vertical height of 9.00 cm opposite the angle of 11.4°.

- Sin 11.4 = opp/hyp = 9/sloping distance.

So, sloping distance = 9/sin 11.4; hence 45.5 to 3 sf. Answer (b) is tan 11.4° = 0.2016 or 0.202 to 3 sf. (c) is 9.0/0.202 = 44.55 cm or 44.6 cm to 3 sf.

- For (d) the triangle is right–angled, so the three sides are related as defined in Pythagoras' theorem: $45.5^2 = 44.6^2 + 9^2$.

Q9 The answers are shown in the completed Table:

angle of slope(α)	gradient (tan α)	% (*x* in 100)	1 in *n*
0.57°	0.01	1.00	1 in 100
1.43°	0.025 0	2.50	1 in 40
4.76°	0.083 3	8.33	1 in 12
5.71°	0.100	10.0	1 in 10
8.14°	0.143	14.3	1 in 7
18.4°	0.333	33.3	1 in 3
26.6°	0.500	50.0	1 in 2
45°	1.00	100	1 in 1

- Neither of the last two examples normally appears on road signs!

- How do you do these conversions? (1) Angle → tangent: just press tan button. Tangent → %: multiply by 100. To convert % → '1 in n': divide the percentage into 100. (2) To go from tan α → angle: press $\tan^{-1}$ button. (3) The only other conversion that can be at all awkward is '1 in n' → %. Do this by dividing the value of 'n' into 100.

Q10 Refer to Figures 19 and 20.

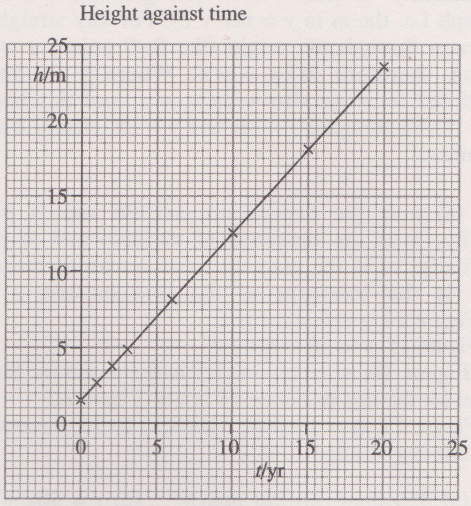


FIGURE 19 Completed Figure 14

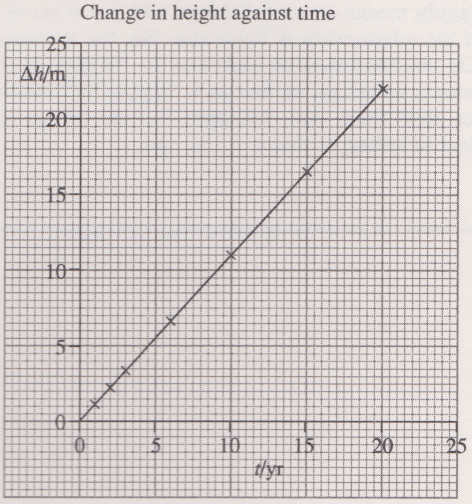


FIGURE 20 Completed Figure 15

(a) The gradients of both are 1.1 m yr^{-1}. Both gradients are the same.
(b) The height of the tree at the time of planting.
(c) $\Delta h \propto t$ is true.
(d) Change in height is directly proportional to the time since planting
(e) $\Delta h = kt$
(f) k = 1.1 m yr^{-1}. This is the same as the gradient.

- The only difference between the graphs in Figures 19 and 20 is that the first cuts the *h* axis at 1.5 m (the height at planting) and the second starts at the origin. The two lines are parallel to each other—their gradients are the same.

- It is the *growth* (Δh) not the *overall height* (h) that is directly proportional to time. This is plain from the graphs and also from simply looking at the Table of data in the question. After one year height is 2.6 m and, after twice the time (two years), the height is 3.7 m—and this is *not* twice the height. However, after one year the growth has been 1.1 m and after two years the growth has been 2.2 m, and so on i.e. 1.1 m per year.

This common sense view is summed up in the words in the answer to (d).

IF YOU FIND THIS MATERIAL DIFFICULT, OMIT PART (f) AND DO NOT LOOK AT THE FOLLOWING BULLETED POINT.

• Part (f) shows how the idea of 'constant of proportionality' is related to m (the gradient of a straight line graph i.e. the m in $y = mx + c$). For any straight line that passes through the origin (the 0, 0 point), $c = 0$. Its equation therefore is $y = mx$. Where y is directly proportional to x, we also write $y \propto x$, and this becomes $y = kx$. If you compare $y = mx$ and $y = kx$, clearly k and m are identical!

Q11 (a) y is the dependent variable and is plotted vertically, x is the independent variable and is plotted horizontally, m is the gradient (also known as the slope), and c is the intercept of the line on the y axis.
(b) In Figure 14, y is h and x is t.
(c) In Figure 15, y is Δh and x is t.
(d) In Figure 14, $m = 1.1\ \text{m yr}^{-1}$ and $c = 1.5$ m. So, the equation for the line in Figure 14 is $y = 1.1x + 1.5$.
(e) In Figure 15, $m = 1.1\ \text{m yr}^{-1}$ and $c = 0$. So, the equation for the line is $y = 1.1x$.

Q12 (a) 350 ppm (b) $50\ \text{ppm h}^{-1}$ (c) $y = 50x + 350$
(d) 800 ppm.

• Notice that you do not need to plot a graph to be able to apply (apply means do useful things with') $y = mx + c$. We are *told* the relationship is linear one. So, the statement that 'the CO_2 concencentration increases by 50 ppm each hour' is equivalent to telling us that $m = 50\ \text{ppm hr}^{-1}$. Similarly, the fact that there are 350 ppm before things start is equivalent to telling us that $c = 350$ ppm.

The equation can then be written down: y and x stay as y and x (the variables) and we substitute the values that we now know for m and c. Thus $50x + 350$.

• Once we have the equation, it is possible it work out any 'y' given a value of x—and, if we wished to, vice versa. Hence, putting x = hours (that is, $t = 9$ hours), algebra gives $50\ \text{ppm} \times 9\ \text{hours} + 350\ \text{ppm} = 800\ \text{ppm}$.

Q13 (a) $20\ \text{ppm min}^{-1}$. This is the value of m.
(b) $y = 550$ ppm and $x = 26$ minutes.
(c) $c = 30$ ppm (initial concentration of chlorine before the leak started).
(d) Equation is $y = 20x + 30$.
(e) 230 ppm.

• In 5 minutes (31 minutes − 26 minutes) the concentration increases by 100 ppm. This is 20 ppm min^{-1}. As the plot is a straight line (given in the question), $m = 20\ \text{ppm min}^{-1}$.

• At $t = 26$ (that is, $x = 26$) we are told that chlorine concentration is 550 ppm (So, $y = 550$).

• These values are now substituted in $y = mx + c$ to give $550 = 20 \times 26 + c$, from which:
$c = 550 - 520 = 30$ ppm. The equation is, therefore, $y = 20x + 30$.

• This equation is used to work out the concentration after 10 minutes by substituting into it $x = 10$ minutes: $y = 20 \times 10 + 30 = 230$ ppm.